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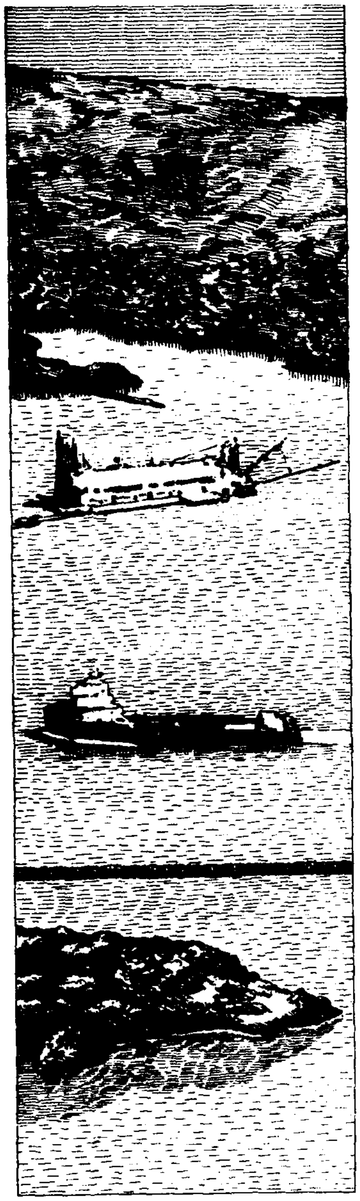

DREDGING RESEARCH PROGRAM

TECHNICAL REPORT DRP-93-2

IMPROVEMENTS TO THE AUTOMATED REAL-TIME TIDAL ELEVATION SYSTEM

by

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DEPARTMENT OF THE ARMY
Waterways Experiment Station, Corps of Engineers
3909 Halls Ferry Road, Vicksburg, Mississippi 39180-6199

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Final Report

Approved for Public Release; Distribution Unlimited

Prepared for DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

Under Work Unit 32478

The Dredging Research Program (DRP) is a seven-year program of the U.S. Army Corps of Engineers. DRP research is managed in these five technical areas:

Area 1 Analysis of Dredged Material Placed in Open Water
Area 2 - Material Properties Related to Navigation and Dredging
Area 3 - Dredge Plant Equipment and Systems Processes
Area 4 - Vessel Positioning, Survey Controls, and Dredge Monitoring Systems
Area 5 - Management of Dredging Projects

US Army Corps
of Engineers
Waterways Experiment
Station

Dredging Research Program
Report Summary

Improvements to the Automated Real-Time Tidal Elevation System, TR DRP-93-2

ISSUE: As presently implemented, the Automated Real-Time Tidal Elevation System (ARTTES) uses a completely deterministic procedure for predicting the tide. When the difference between the predicted tide and the observed level exceeds some predetermined value, user access to the system is denied. Periods of user-access denial usually occur when sea conditions are unfavorable or unsuitable for survey operations. Dredging operations, however, may continue in weather conditions unsuitable for survey operations. Because ARTTES is sometimes used by dredge operators, it is desirable to minimize periods of user-access denial.

RESEARCH: A hybrid approach was used to improve the ARTTES tide predictions: "hybrid" in that the astronomically forced component of the tide was analyzed and predicted in the traditional manner via harmonic analysis while the short-term meteorological component (the residual) was analyzed and predicted via statistical techniques. Kriging and autoregression statistical techniques were investigated.

SUMMARY: The two statistical techniques were applied to improve short-term tide prediction. Evaluations using both synthetic and prototype data indicated that although Kriging has a slightly larger root mean square error than the autoregression technique, it gives an unbiased estimate and the autoregression technique shows bias.

Because dredges are costly to operate, data-access denial should only occur when the potential consequences warrant. The short-term tide predictions provide an objective basis for a decision by the dredging supervisor to use or ignore the ARTTES during periods of degraded data.

AVAILABILITY OF REPORT: The report is available through the Interlibrary Loan Service from the U.S. Army Engineer Waterways Experiment Station (WES) Library, telephone number (601) 634-2355. National Technical Information Service report numbers may be requested from WES Librarians. To purchase a copy of the report, call NTIS at (703) 487-4780.

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Dredging Research Program

Technical Report DRP-93-2
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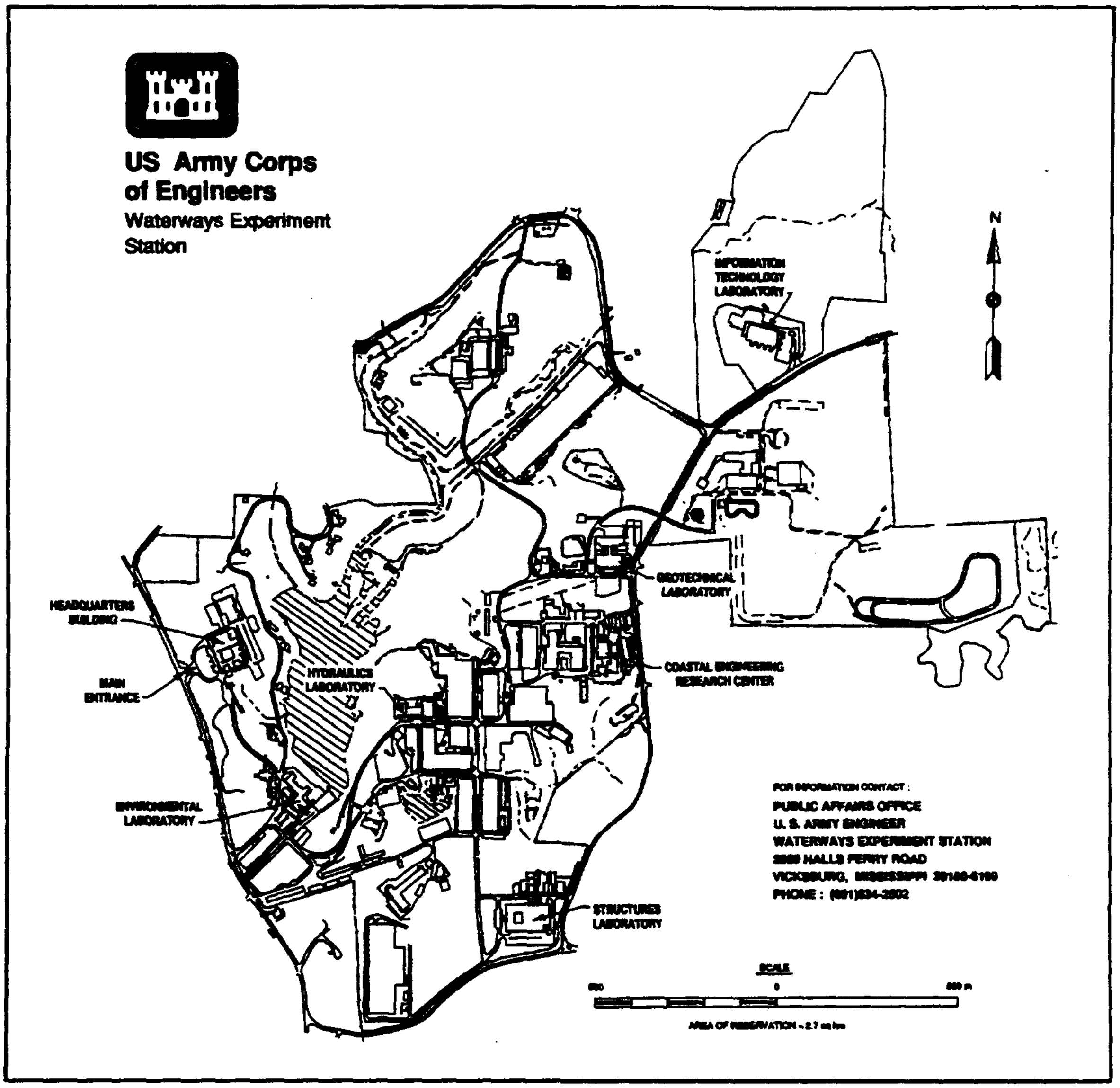

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Contents

Preface

The work described herein was authorized as part of the Dredging Research Program (DRP) of Headquarters, U.S. Army Corps of Engineers (HQUSACE), and performed under the Integrated Vertical Control System Work Unit 32478, which is part of DRP Technical Area 4, Vessel Positioning, Survey Controls, and Dredge Monitoring Systems. Messrs. Robert Campbell and Moody K. Miles III were DRP Chief and Technical Area 4 Monitor, respectively, from HQUSACE. Mr. E. Clark McNair, Jr., Coastal Engineering Research Center (CERC), U.S. Army Engineer Waterways Experiment Station (WES), was DRP Program Manager (PM), and Dr. Lyndell Z. Hales, CFRC, was Assistant PM. Dr. Andrew W. Garcia, CERC, and Mr. George P. Bonner, Chief, WES Instrumentation Services Division, were Technical Managers for Technical Area 4 during the conduct of this study. This study was performed and the report prepared over the period October 1990 to June 1992 by Drs. Leon E. Borgman, L. E. Borgman, Inc., Todd L. Walton, CERC, and Andrew W. Garcia, CERC. Dr. Garcia was under the administrative supervision of Mr. William L. Preslan, Chief, Prototype Measurement and Analysis Branch, Engineering Development Division, CERC. Dr. Walton was under the administrative supervision of Mr. Thomas W. Richardson, Chief, Engineering Development Division, CERC. Dr. James R. Houston was Director, CERC; Mr. Charles C. Calhoun, Jr., was Assistant Director, CERC.

Dr. Robert W. Whalin was Director of WES during the preparation of this report. COL Bruce K. Howard, EN, was Commander.

1 Introduction

Procedures for increasing the availability of real-time tide data broadcast by Automated Real-Time Tidal Elevation Systems (ARTTES) (Lillycrop et al. 1991) are described herein. The measured water-level data, which are broadcast to user vessels, are compared to the predicted tide at the water-level sensor. As presently implemented, ARTTES uses a completely deterministic procedure for predicting the tide. When the difference between the predicted tide and observed level exceeds some predetermined value, user access to the system is denied. The difference between measured and predicted values (at the measurement site) is an indicator of the validity of the assumption that the astronomical tide component is dominant. Dominance is defined herein to mean that the astronomical component of the water-level variation is approximately one order of magnitude greater than the sum of the remaining components. Zero or small (a few centimeters) differences mean that the assumption is robustly met within the entire specified area of operation and that users can be confident of the broadcast system values. When differences between the predicted and observed values increase, as is typically the case during periods of disturbed weather, the assumption of dominance of the astronomical tide component grows increasingly weak. At some difference between the predicted tide and observed water level, the assumption of tidal dominance is considered to be invalid for the desired degree of water-level accuracy over the area of operation.

Periods of user-access denial usually occur when sea conditions are unfavorable or unsuitable for survey operations. Dredging operations, however, may continue in weather conditions unsuitable for survey operations. Because ARTTES is sometimes used by dredge operators, it is desirable to minimize periods of user-access denial.

The present procedure for predicting the tide at the sensor location is based upon the harmonic method of tide analysis and prediction (Schureman 1985). Water-level data at the desired location are acquired for several months, analyzed to determine the amplitude and phase of the dominant tidal constituents, then used to predict the tide. The data required for tide prediction are obtained prior to initial operation of an ARTTES. Although water-level data at the sensor site are recorded and available once an ARTTES begins operation, analysis of these data by the harmonic method and incorporation of the results into the prediction routine presently being used typically result in little improvement in the predictions. The lack of improvement in tide prediction

stems from the fundamental assumption underlying the harmonic method, namely that the tide is described by a sum of sine or cosine functions that represent the astronomically forced component (Godin 1972). However, a portion of the water-level record is not described by this sum. That portion is termed the residual. Within the context of this report, most of the residual is the result of short-term meteorological effects.

The approach used in improving the ARTTES tide predictions may be described as "hybrid" in that the astronomically forced component of the tide is analyzed and predicted in the traditional manner (via harmonic analysis), while the short-term meteorological component (the residual) is analyzed and predicted via statistical techniques. The statistical techniques investigated are conventionally called "Kriging" and autoregression. Because tidal harmonic analysis is well-documented and its use well-established (e.g., Godin 1972; Schureman 1985), no further description is contained herein. The chapters in this report are arranged to describe (a) the problem statement, (b) an overview of the Kriging and autoregressive methods, (c) implementation of the methods, (d) results, including a case application, and (e) a summary of conclusions. The Appendices document one of the case applications. Appendix A is a synthesized time series of known statistical properties that simulates a residual time series. Appendix B is a listing of the computer program used to prepare both Kriged and autoregessive estimates of the residual. Appendix C is a tabulation of the computed results using the Kriging estimate method.

2 Background

If z(t) represents the observed water level at any time t, the water level z, including the harmonic representation of the tide, may be written as

$$z(t) = z_0 + \sum_{k=1}^{H} A_k \cos(\omega_k t - a_k) + V(t) \tag{1}$$

where

z_0 = mean water level (or other datum)

H = number of significant harmonic components

A_k, a_k = amplitude and phase lag, respectively, of kth component

ω_k = frequency present in tide-generating force

$V(t)$ = contribution of physical forcing other than astronomical

The residual is then

$$V(t) = z(t) - z_0 - \sum_{k=1}^{H} A_k \cos\left(\omega_k t - a_k\right) \tag{2}$$

The observed water level $z(t)$ is normally logged by the ARTTES at 6-min intervals (10 values per hour). The residual $V(t)$, the time series upon which the statistical procedure operates, is obtained by subtracting the predicted tide at concurrent times.

Consider a discrete time series of residual values

$$\left\{V_i;\ i = 1,2,3,\ldots,N\right\} \tag{3}$$

Having

$$E[V_i] = \mu_i$$
$$E[(V_i - \mu_i)(V_{i+k} - \mu_{i+k})] = C_k \quad (4)$$

where

$E[\ ]$ = is the expected value of the quantity in brackets

μ_i = mean of series V_i

C_k = covariance of series V_i at log k

That is,

$$W_i = V_i - \mu_i \quad (5)$$

is taken as being covariance stationary with mean zero, where W_i is the residual with its mean removed.

The purpose of this study is to determine the values of $V_n, V_{n+1}, V_{n+2}, \ldots, V_{n+p-1}$ from measured values of $(V_{n-1}, V_{n-2}, V_{n-3}, \ldots, V_{n-m})$.

The data values $\{V_i;\ n-m-L \le i \le n-1\}$ for a user-selected extension $L \ge 0$, will be fitted with a straight line to estimate μ_i as a linear trend

$$\hat{\mu}_i = a + bi \quad (6)$$

where

$\hat{\mu}$ = is the estimated local mean

a, b = parameters calculated at each time-step

$\hat{}$ = a parameter to be estimated

The covariance function will be estimated as mean lag products of deviations from the local mean

$$\hat{C}_k = \frac{1}{m+L-k} \sum_{i=n-m-L}^{n-k-1} \{V_i - a - bi\} \{V_{i+k} - a - b(i+k)\} \quad (7)$$

The correlation function will then be obtained

$$\hat{\rho}_K = \hat{C}_K / \hat{C}_0 \tag{8}$$

3 Overview of Methods

Two methods were used to improve the ARTTES tide predictions: Kriging and autoregressive estimation. Kriging is based on the minimization of expected square error, subject to the constraint that the estimation is unbiased. Autoregression is based on minimization of expected squared error only. Both methods will assume that the estimate is covariance-stationary.

Kriging Procedure

Here, $\hat{V}_{n+k}$ is the estimate of the residual at time-step $n+k$ and is predicted as a linear combination of the data for time-steps preceding time-step n.

$$\hat{V}_{n+k} = \sum_{i=1}^{m} a_i V_{n-m+i-1} \tag{9}$$

So as to minimize Q, the expected value of the error squared between the actual and estimated residuals

where

$$Q = E\left[\left(V_{n+k} - \hat{V}_{n+k}\right)^2\right] \tag{10}$$

Subject to the unbiased constraint

$$E\left[V_{n+k} - \hat{V}_{n+k}\right] = 0 \tag{11}$$

which requires that the expected value of the estimator be equal to the expected value of the residual at time step $n+k$

Since,

$$\mu_i = E\left[V_i\right] \tag{12}$$

the "unbiased" constraint can be written

$$\mu_{n+k} - \sum_{i=1}^{m} a_i \mu_{n-m+i-1} = 0 \tag{13}$$

or,

$$\sum_{i=1}^{m} a_i \mu_{n-m+i-1} = \mu_{n+k} \tag{14}$$

Similarly, Q can be expanded to

$$Q = E\left[\left[V_{n+k} - \sum_{i=1}^{m} a_i V_{n-m+i-1}\right]^2\right] \tag{15}$$

Since the "unbiased" conditions are going to be imposed, the expression does not change if the "unbiased" constraint equation (equal to zero) is inserted inside the square

$$Q = E\left[\left\{\left(V_{n+k} - \mu_{n+k}\right) - \sum_{i=1}^{m} a_i\left(V_{n-m+i-1} - \mu_{n-m+i-1}\right)\right\}^2\right] \tag{16}$$

If the algebra of squaring is completed

$$Q = E\left[\left(V_{n+k} - \mu_{n+k}\right)^2 + \left\{\sum_{i=1}^{m} a_i \left(V_{n-m+i-1} - \mu_{n-m+i-1}\right)\right\}^2 - 2\sum_{i=1}^{m} a_i \left(V_{n+k} - \mu_{n+k}\right)\left(V_{n-m+i-1} - \mu_{n-m+i-1}\right)\right] \tag{17}$$

The term with the summation sign Σ squared can be expanded as follows:

$$\left\{\sum_{i=1}^{m} a_i\left(V_{n-m+i-1}-\mu_{n-m+i-1}\right)\right\}\left\{\sum_{j=1}^{m} a_j\left(V_{n-m+j-1}-\mu_{n-m+j-1}\right)\right\} \tag{18}$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{n} a_i a_j\left(V_{n-m+i-1}-\mu_{n-m+i-1}\right)\left(V_{n-m+j-1}-\mu_{n-m+j-1}\right)$$

Hence, the expected square error term "Q" may be expressed as

$$Q = E\left[\left(V_{n+k}-\mu_{n+k}\right)^2\right]$$
$$+ \sum_{i=1}^{m}\sum_{j=1}^{n} a_i a_j E\left[\left(V_{n-m+i-1}-\mu_{n-m+i-1}\right)\left(V_{n-m+j-1}-\mu_{n-m+j-1}\right)\right] \tag{19}$$
$$- 2\sum_{i=1}^{m} a_i E\left[\left(V_{n+k}-\mu_{n+k}\right)\left(V_{n-m+i-1}-\mu_{n-m+i-1}\right)\right]$$

By definition of the covariance function, and the assumption of covariance stationarity after removing the trend

$$C_k = Cov\left(V_i,\ V_{i+k}\right) = E\left[\left(V_i-\mu_i\right)\left(V_{i+k}-\mu_{i+k}\right)\right] \tag{20}$$

It follows that

$$Q = C_o + \sum_{i=1}^{m}\sum_{j=1} a_i a_j C_{i-j} - 2\sum_{i=1}^{m} a_i C_{k+m-i+1} \tag{21}$$

The determination of the coefficients $\{a_i\}$ for Kriging then involves solving for the $\{a_i\}$, which minimize Q subject to the constraint of being unbiased.

By the method of Lagrangian multipliers, this is achieved by solving the system of equations given by

$$\frac{\partial}{\partial a_s}\left[Q - 2\lambda\left(\mu_{m+k} - \sum_{i=1}^{m} a_i\mu_{n-m+i-1}\right)\right] = 0$$

$$s = 1,2,3,\ldots m \tag{22}$$

and by the *unbiased* constraint Equation (13)

$$\sum_{i=1}^{m} a_i\mu_{n-m+i-1} = \mu_{n+k}$$

Performing the differentiation gives

$$2\sum_{j=1}^{m} a_j C_{s-j} - 2\,C_{k+m-s+1} = 2\lambda\mu_{n-m+s-1} = 0 \tag{23}$$

If the equations are listed for $s = 1,2,3,\ldots,m,$, using $C_k = C_k$, they become

$$\begin{aligned}
C_0a_1 = C_1a_2 + \ldots + C_{m-1}a_m\,\lambda\mu_{n-m} &= C_{k+m}\\
C_1a_1 + C_0a_2 + \ldots + C_{m-2}a_m + \lambda\mu_{n-m-1} &= C_{k+m-1}\\
&\ \vdots\\
C_{m-1}a_1 + C_{m-2}a_2 + \ldots + C_0a_m + \lambda\mu_{n-1} &= C_{k+1}\\
\mu_{n-m}a_1 + \mu_{n-m+1}a_2 + \ldots + \mu_{n-1}a_m &= \mu_{n+k}
\end{aligned} \tag{24}$$

The first m equations can be divided by C_0 to express the relations in terms of the correlation coefficients ρ_k

$$\begin{aligned}
a_1 + \rho_1a_2 + \ldots + \rho_{m-1}a_m = (\lambda/C_0)\,\mu_{n-m} &= \rho_{k+m}\\
\rho_1a_1 + a_2 + \ldots + \rho_{m-2}a_m + (\lambda/C_0)\,\mu_{n-1} &= \rho_{k+m-1}\\
&\ \vdots\\
\rho_{m-1}a_1 + \rho_{m-2}a_2 + \ldots + a_m + (\lambda/C_0)\mu_{n-1} &= \rho_{k+1}\\
\mu_{n-m}a_1 + \mu_{n-m+1}a_2 + \ldots + \mu_{n-1}a_m &= \mu_{n+k}
\end{aligned} \tag{25}$$

This can be written in matrix form as

$$\begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{m-1} & \mu_{n-m} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{m-2} & \mu_{n-m+1} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{m-3} & \mu_{n-m+2} \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \rho_{m-1} & \rho_{m-2} & \rho_{m-3} & \cdots & 1 & \mu_{n-1} \\ \mu_{n-m} & \mu_{n-m+1} & \mu_{n-m+2} & \cdots & \mu_{n-1} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \cdot \\ \cdot \\ \cdot \\ a_m \\ \lambda/C_0 \end{bmatrix} = \begin{bmatrix} \rho_{k+m} \\ \rho_{k+m-1} \\ \rho_{k+m-2} \\ \cdot \\ \cdot \\ \cdot \\ \rho_{k+1} \\ \mu_{n+k} \end{bmatrix} \tag{26}$$

This can be further simplified if the following definitions are introduced

$$R = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{m-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{m-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{m-3} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \rho_{m-1} & \rho_{m-2} & \rho_{m-3} & \cdots & 1 \end{bmatrix} \tag{27}$$

$$\rho_0 = \begin{bmatrix} \rho_{k+m} \\ \rho_{k+m-1} \\ \rho_{k+m-2} \\ \cdot \\ \cdot \\ \cdot \\ \rho_{k+1} \end{bmatrix} \tag{28}$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \cdot \\ \cdot \\ \cdot \\ a_m \end{bmatrix} \tag{29}$$

$$\mu = \begin{bmatrix} \mu_{n-m} \\ \mu_{n-m+1} \\ \mu_{n-m+2} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{n-1} \end{bmatrix} \tag{30}$$

The Kriging equations may be written in terms of these arrays as

$$\begin{bmatrix} R & \mu \\ \mu^T & 0 \end{bmatrix} \begin{bmatrix} a \\ \lambda/C_0 \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \mu_{n+k} \end{bmatrix} \tag{31}$$

The mean-square error of estimate is the value of Q with the a_j, which are solutions to the Kriging equations inserted. If (21) is multiplied by a_s and summed over s, the result is (dividing by 2)

$$\sum_{\ell=1}^{m} \sum_{j=1}^{m} a_\ell a_j C_{\ell-j} - \sum_{\ell=1}^{m} a_\ell C_{k+m-\ell+1} + \lambda \sum_{\ell=1}^{m} a_\ell \mu_{n-m+\ell-1} = 0 \tag{32}$$

By the "unbiased" constraint, the summation involved $\mu_{n-m+\ell-1}$ may be written as μ_{n+k}. Consequently, the last equation becomes

$$\sum_{\ell=1}^{m} \sum_{j=1}^{m} a_\ell a_j C_{\ell-j} = \sum_{\ell=1}^{m} a_\ell C_{k+m-\ell+1} - \lambda\mu_{n+k} \tag{33}$$

This relation holds for solutions to the Kriging equations. If this relation is substituted into Q,

$$Q = C_0 - \sum_{i=1}^{m} a_i C_{k+m-i+1} - \lambda\mu_{n+k} \tag{34}$$

The root-mean-square (rms) error of estimate for the Kriging method presented is

$$\hat{\sigma}_\epsilon = \sqrt{C_0 - \sum_{i=1}^{m} a_i C_{k+m-i+1} - \lambda\mu_{n+k}} \tag{35}$$

where the $\{a_i\}$ are the solutions to the Kriging equations.

Autoregressive Procedure

The autoregressive technique is based on the assumed structural model equation that, for all j,

$$W_j = \sum_{i=1}^{m} a_i W_{j-m+i-1} + \epsilon_j \tag{36}$$

with W_j as defined previously as per Equation 5 and

where the sequence $\{\epsilon_j\}$ is taken independently, with mean zero and constant variance. The $\{a_i\}$ are parameters to be fit via the methodology to be presented below. This means that with the stationary condition, for $\ell < j$

$$\begin{aligned} C_j - l &= E[W_\ell W_j] \\ &= E\left[W_\ell \left\{\sum_{i=1}^{m} a_i W_{j-m+i-1} + \epsilon_j\right\}\right] \end{aligned} \tag{37}$$

Since $\ell < j$, the error in W_j is independent of W_ℓ and $E[W_\ell \epsilon_j] = 0$. Consequently,

$$\begin{aligned} C_{j-\ell} &= \sum_{i=1}^{m} a_i E[W_\ell W_{j-m+i-1}] \\ &= \sum_{i=1}^{m} a_i C_{j-\ell-m+i-1} \end{aligned} \tag{38}$$

The model equation for W_n is

$$W_n = a_1 W_{n-m} + a_2 W_{n-m+1} + a_3 W_{n-m+2} + \ldots + a_m W_{m-1} + \epsilon_n \tag{39}$$

If $j = n$ while $n\text{-}m \leq \ell < n$, the covariance becomes

$$\begin{aligned} C_{n-\ell} &= \sum_{i=1}^{m} a_i C_{n-\ell-m+i-1} \\ &= a_1 C_{n-m-\ell} + a_2 C_{n-m-\ell+1} + a_3 C_{n-m-\ell+2} + \ldots + a_m C_{n-\ell-1} \end{aligned} \tag{40}$$

Now, if successively, $\ell = n\text{-}m, n\text{-}m+1, n\text{-}m+2, \ldots., n\text{-}1$, the resulting system of equations is (using $C_{-j} = C_j$)

$$a_1C_0+a_2C_1+a_3C_2+..+a_mC_{m-1} = C_m$$
$$a_1C_1+a_2C_0+a_3C_1+..+a_mC_{m-2} = C_{m-1}$$
$$a_1C_2+a_2C_1+a_3C_0+..+a_mC_{m-3} = C_{m-2} \qquad (41)$$
$$\vdots$$
$$a_1C_{m-1}+a_2C_{m-2}+a_3C_{m-3}+..+a_mC_0 = C_1$$

The system of equations can be expressed in matrix form, after division with C_0, as

$$\begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{m-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{m-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{m-3} \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ \rho_{m-1} & \rho_{m-2} & \rho_{m-3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ . \\ a_m \end{bmatrix} = \begin{bmatrix} \rho_m \\ \rho_{m-1} \\ \rho_{m-2} \\ . \\ . \\ . \\ \rho_1 \end{bmatrix} \qquad (42)$$

This is one form of the well-known Yule-Walker equations for the coefficients $\{a_i\}$ to predict W_n from $\{W_j;\ n\text{-}m \le j \le n\text{-}1\}$ in a stationary sequence. It is convenient to express the prediction of W_n in a somewhat redundant way.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ . & . & . & . & & . \\ . & . & . & . & & . \\ . & . & . & . & & . \\ 0 & 0 & 0 & 0 & \ldots & 1 \\ a_1 & a_2 & a_3 & a_4 & \ldots & a_m \end{bmatrix} \begin{bmatrix} W_{n-m} \\ W_{n-m+1} \\ W_{n-m+2} \\ . \\ . \\ . \\ W_{n-2} \\ W_{n-1} \end{bmatrix} = \begin{bmatrix} W_{n-m+1} \\ W_{n-m+2} \\ W_{n-m+3} \\ . \\ . \\ . \\ W_{n-1} \\ W_n \end{bmatrix} \qquad (43)$$

This can be written as

$$A\mathbf{W}_{n-1} = \mathbf{W}_n \qquad (44)$$

where A is the matrix, and the bold print denotes vectors. The actual prediction of W_n is given by the last component of the vector $\mathbf{W}_n$. The matrix A is a shift operator that moves the sequence up by one step. The prediction of W_{n+1} is the last component of $\mathbf{W}_{n+1}$, where

$$W_{n+1} = AW_n = A^2W_{n-1} \tag{45}$$

Similarly,

$$\begin{aligned} W_{n+2} &= AW_{n+1} = A^3W_{n-1} \\ &\vdots \\ W_{n+k} &= AW_{n+k-1} = A^{k+1}W_{n-1} \end{aligned} \tag{46}$$

Alternatively, this sequence of equations can be used to predict $\{W_n, W_{n+1}, W_{n+2}, \ldots, W_{n+k}\}$ from $\{W_{n-m}, W_{n-m+1}, W_{n-m+2}, \ldots, W_{n-1}\}$. Also, the mean square error of estimate for each of these predictions can be obtained from a modification of this same sequence of equations. Since $E[W_j] = 0$ (by subtraction of trend), if C_{n+k} is defined as the covariance matrix of W_{n+k},

$$C_{n+k} = E\left[W_{n+k}W_{n+k}^T\right] \tag{47}$$

Here, the superscript "T" denotes the matrix transpose. It follows that

$$\begin{aligned} C_{n+k} &= E\left[(AW_{n+k-1})(AW_{n+k-1})^T\right] \\ &= AE\left[W_{n+k-1}W_{n+k-1}^T\right]A^T \\ &= AC_{n+k-1}A^T \end{aligned} \tag{48}$$

for $k = 0,1,2,3,\ldots$. The variance of the prediction of W_{n+k} is the (m,m) - element of C_{n+k}. Since

$$C_{n-1} = \begin{bmatrix} C_0 & C_1 & C_2 & \cdots & C_{m-1} \\ C_1 & C_0 & C_1 & \cdots & C_{m-2} \\ C_2 & C_1 & C_0 & \cdots & C_{m-3} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ C_{m-1} & C_{m-2} & C_{m-3} & \cdots & C_0 \end{bmatrix} \tag{49}$$

the variance of each prediction can be obtained as the (m,m) - element of each of the sequence of covariance matrices

$$
\begin{aligned}
C_n &= A\,C_{n-1}A^T \\
C_{n+1} &= A\,C_nA^T \\
C_{n+2} &= A\,C_{n+1}A^T \\
&\ \ \vdots \\
C_{n+k} &= A\,C_{n+k-1}A^T
\end{aligned}
\tag{50}
$$

An algorithm that takes advantage of the structure of A where

$$
A = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & 0 & \dots & 1 \\ a_1 & a_2 & a_3 & a_4 & \dots & a_m \end{bmatrix} = \begin{bmatrix} 0 & I \\ a_1 & A^T \end{bmatrix}
\tag{51}
$$

where **0** is an (m-1) - component vector of zeros, I is an $(m-1)$ by $(m-1)$ identity matrix, and

$$
A = \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ \cdot \\ \cdot \\ \cdot \\ a_m \end{bmatrix}
\tag{52}
$$

Let a general step in the iterative computation of $\{C_{n+k};\ k = 0,1,2,3,\dots\}$ be represented by

$$
B = ACA^{\mathrm{T}}
\tag{53}
$$

Furthermore, let C be partitioned as

$$C = \begin{bmatrix} C_{11} & C^T \\ C & C_{22} \end{bmatrix} \tag{54}$$

where C_{11} is a scalar, C_{22} is an $(m\text{-}1)$ by $(m\text{-}1)$ matrix, and C is an $(m\text{-}1)$ - component column vector.

Then, similarly, let B be partitioned as

$$B = \begin{bmatrix} B_{11} & B \\ B^T & B_{22} \end{bmatrix} \tag{55}$$

where B_{11} is an $(m-1)$ by $(m-1)$ matrix, B is an $(m-1)$ - component column vector, and B_{22} is a scalar. Then,

$$\begin{bmatrix} B_{11} & B \\ B^T & B_{22} \end{bmatrix} = \begin{bmatrix} 0 & I \\ a_1 & A^T \end{bmatrix} \begin{bmatrix} C_{11} & C^T \\ C & C_{22} \end{bmatrix} \begin{bmatrix} 0^T & a_1 \\ I & A \end{bmatrix} \tag{56}$$

It follows that

$$\begin{aligned} B_{11} &= C_{22} \\ B &= a_1 C + C_{22} A \\ B_{22} &= C_{11} a_1^2 + 2a_1 A^T C + A^T C_{22} A \end{aligned} \tag{57}$$

(Note: The scalar B_{22} is the variance of the prediction at that particular iteration.)

The algorithm consists of:

(1) Compute $a_1 C$ and store in B.

(2) Multiply B by 2.0 and store in work vector D.

(3) Compute vector $C_{22}A$ and add into both B and D.

(4) Calculate the scalar $A^T D$ and store in B_{22}.

(6) Move C_{22} into upper left (m-1) x (m-1) submatrix of new covariance matrix.

(7) Store B into upper right (m-1) by 1 submatrix (i.e., vector) of new covariance, and its transposition into the lower left 1 by (m-1) row vector.

(8) Store B_{22} in the (m,m) position of the new covariance matrix.

This sequence of computations will allow the new covariance matrix to be stored into the same memory as the preceding covariance matrix. The rms estimate of error of prediction is given by

$$\hat{\sigma}_{AR} = C_0 + \sum_{i=1}^{m} \sum_{j=1}^{m} a_i a_j C_{i-j} - 2\sum_{i=1}^{m} a_i C_{m-i+1} \tag{58}$$

4 Implementation

The Low-Pass Filter

An initial low-pass filter is applied to the time series to eliminate any high-frequency noise that may remain in the signal after its earlier processing. The time series for times from J0-LPREV to J0+M-1 (i.e., M+LPREV = MPLP time-steps) is first transformed with the fast Fourier transform (FFT) algorithm to the frequency domain. An FFT subroutine based on MPLP time-steps being expressible as a product of powers of 2, 3, and 5 is used. The frequency-domain version of the time series interval is then multiplied, for each frequency step, by a filter function that is almost 1.0 for frequencies between -FLPASS and +FLPASS, and almost zero for frequencies with absolute value greater than FLPASS+WIDTH. A smooth transition within the interval [FLPASS, FLPASS+WIDTH], and its mirror image in negative frequencies is achieved with a cumulative normal probability shape. The accuracy with which 1.0 on low frequencies and 0.0 on high frequencies is attained is controlled with the input variable FRAC in the subroutine call. If FRAC = 0.01, for example, the error will be less than 1 percent. Since the frequency filter function is symmetric about frequency equal zero, the filter will have zero phase shift. All frequencies (FLPASS and WIDTH) are expressed in cycles per time unit, rather than time-step number. Thus, these values can be entered independently of the number of steps in the FFT call.

The Computer Program

The computer program PREDICT.FOR assumes input of a time series, $\{V_j\}$, for $j = 1,2,3,\ldots,$ NJ as read from file TIMESER.DAT. A maximum value for m is specified by M, although the program may choose to use a shorter m value than M. A constant JB is input in parameter statements, which controls the time-step for the first prediction, at step JB+M. Estimates are then made for each time-step from JB+M to the end of the sequence. An rms error of estimate is provided at each step.

After the application of the low-pass filter on the interval [J0-LPREV, J0+M-1] at each prediction, a straight line is fitted to the data for time-steps

$$J0 - LPREV \le j \le J0 + M - 1 \tag{59}$$

where the prediction is being made for step $J0 + M$. This straight line is subtracted out

$$W_j = V_j - a - bj \tag{60}$$

and covariance function estimates are made over the same interval of time-steps.

The computer program searches for the lag just before the lag at which the covariance function has its first down-crossing of zero. The covariance function at all higher lags is set to zero, and m is set to equal the terminating lag. This seems to give good estimates that can be obtained quickly on the computer. However, the code is easy to adjust, should other choices appear better in subsequent use.

The parameter IFLAG controls which prediction method is used. If IFLAG is 1, then only Kriging estimates are prepared. If IFLAG = 2, only the autoregressive methods are used. If IFLAG = 3, both procedures are calculated. Several other flags are included in the program for the convenience of the user. The flag IDOC causes a variety of check output to be printed on DEVICE 2 if IDOC=1. If IDOC=0 the check output is suppressed. Another flag, IBYPASS, controls whether or not detailed prediction output is printed into DEVICE 2. If IBYPASS=0, the material is printed (i.e., the output section is not bypassed). If IBYPASS=1, the section is bypassed. A brief listing of the low-pass filtered series and the shutdown decisions is output onto DEVICE 4.

Three appendices at the end of this report list, respectively, the input file GARSHOR.DAT (in compressed form), the computer program PREDICT6.for, and a lengthy listing of the output results from DEVICE 4.

In the output, the autoregressive estimates have slightly less rms error, but are biased, with a bias of

$$\begin{aligned} \frac{bias}{true\ value} &= 1 - SUM\ AR\ COEF = 1 - 0.80226 \\ &= 0.19774 \end{aligned} \tag{61}$$

The Kriging estimates are constrained to be unbiased, but as a result, they have a slightly larger rms error. Both estimates appear good and involve about the same computational difficulty.

Filter Characteristics

Filter characteristics were studied by an approximate procedure. Since both the Kriging and the autoregressive approaches depend on the actual multiplier coefficients and the covariance function of the input data, general results cannot be given. However, approximate behavior can be deduced from an assumption that the coefficients produce behavior about like a convolution with a half-tent function given in the log domain as

$$w(h) = \begin{cases} \frac{2}{h_0}\left\{1 - \frac{h}{h_0}\right\}, & 0 \le h \le h_0 \\ 0 \quad , & otherwise \end{cases} \tag{62}$$

where

h_o = the function width

The Fourier transform of $w(h)$ and its associated amplification and phase functions represent an approximate description of the filter characteristics. The Fourier transform of $w(h)$ is

$$W(f) = \int_0^{h_0} \frac{2}{h_0}\left\{1 - \frac{h}{h_0}\right\} e^{-i2\pi fh} dh \tag{63}$$

After integration by parts and some algebra, this reduces to

$$W(f) = \left[1 - e^{-i\pi fh_0}\frac{\sin(\pi fh_0)}{\pi fh_0}\right]/(i\pi fh_0) \tag{64}$$

The real and imaginary parts of $W(f)$ are

$$Re\{W(f)\} = \left[\frac{\sin(\pi fh_0)}{\pi fh_0}\right]^2 \tag{65}$$

$$Im\{W(f)\} = -\left[\frac{1}{\pi fh_0} - \frac{\sin(\pi fh_0)\cos(\pi fh_0)}{(\pi fh_0)^2}\right] \tag{66}$$

The amplification and phase functions of the filter are then

$$AMP(f) = \sqrt{[Re\{W(f)\}]^2 + [Im\{W(f)\}]^2} \tag{67}$$

$$PHAZ(f) = \arctan[Im\{W(f)\}/Re\{W(f)\}] \tag{68}$$

All of these are functions of fh_0. Hence, a table of frequency characteristics of the half-tent filter can be listed versus values of fh_0, which is dimensionless. The filter has an amplification of 1.0 at $f = 0$, and a half-peak drop-off at $fh_0 = 0.7745$. Since h_0 in the data used is around 8, the half-peak frequency would be 0.7745/8 = 0.097 cycles per time-step, or a corresponding wave length of 10 time-steps. Wavelengths shorter than this are being suppressed by the filter.

Stationarity of Prediction Sequence

The sequence of predictions $\{\hat{V}_j, j=n,n+1,n+2,...\}$, as based on $\{V_i, i=n-m,n-m+1,n-m+2,...,n-1\}$, is not a statistically stationary sequence in either Kriging or autoregressive estimation. The rms error varies with time-step. There are other differences depending on the specific properties of each method. It is particularly revealing to examine the asymptotic behavior of the sequences for each method.

Asymptotic Behavior of Kriging

The basic structure of the Kriging estimation is given by Equation 31. If the time-step at the estimation is sufficiently far ahead of the data time-steps used in making the estimate that the value there is uncorrelated with the data, then $\boldsymbol{\rho}_0 = \mathbf{0}$, and the solution to Equation 31 can be written as

$$b = \mu^T R^{-1} \mu$$

$$\begin{bmatrix} a \\ \lambda/C_0 \end{bmatrix} = \begin{bmatrix} R^{-1}[I - \mu\mu^T R^{-1}/b] & R^{-1}\mu/b \\ \mu^T R^{-1}/b & -1.0/b \end{bmatrix} \begin{bmatrix} 0 \\ \mu_{n+k} \end{bmatrix} \tag{69}$$

Hence, the Kriging coefficients and Lagrangian coefficient are

$$\begin{aligned} a &= \mu_{n+k} R^{-1}\mu/b \\ \lambda &= \mu_{n+k} C_0/b \end{aligned} \tag{70}$$

and the prediction and mean-square-error is

$$\hat{V}_{n+k} = \mu_{n+k} \frac{V^T R^{-1} \mu}{\mu^T R^{-1} \mu}$$

$$\sigma_E^2 = C_0 \left[1.0 - \frac{\mu_{n+k}}{\mu^T R^{-1} \mu} \right] \tag{71}$$

Consequently, the Kriging error does not go to zero, but has some value that depends on R. The asymptotic estimate of V converges to a weighted average of the data values times the mean at the prediction time-step.

Asymptotic Behavior for the Autoregressive Estimates

From Equation 46, asymptotic behavior of W_{n+k} depends on the absolute value of the largest eigenvalue of the matrix A. This can be less than 1.0, equal to 1.0, or greater than 1.0 for reasonable choices of correlation coefficients. (For example, in an m=2 autoregressive sequence with correlation coefficient at lag 1 equal to 0.5 and at lag 2 equal to 0.1, the absolute value of the largest eigenvalue is 0.4472. If the lag 1 correlation coefficient is changed to 0.9 while the lag 2 correlation coefficient is left at 0.1, the absolute value of the largest eigenvalue is 1.2247. If the absolute value of the eigenvalue is less than 1.0, then the asymptotic value of the estimate of W_{n+k} will tend towards zero as k increases indefinitely. Similarly, if the absolute value of the largest eigenvalue is greater than 1.0, then the estimate will become indefinitely large as k increases.

5 Results and Conclusions

Case Applications

Application of the Kriging or autoregressive procedures requires the user to specify two parameters related to the physical situation. The first is the threshold value of the difference between observed and predicted water levels (the residual) beyond which ARTTES data may be degraded (variable THOLD in Appendix B). The second is the permissible duration of the threshold exceedance (variable NPF in Appendix B). The threshold value is chosen using engineering judgment and knowledge of residual values from previous water-level monitoring at the site. The goal is to choose a value such that smaller residuals represent meteorological effects that are large enough in scale to have a uniform effect over the specified area of ARTTES operation. The assumption of spatial uniformity becomes somewhat questionable for residuals exceeding the threshold.

In the earlier non-adaptive procedure, user access was denied whenever the residual exceeded a preset limit, even if the threshold was just slightly exceeded. The Kriging or autoregressive procedures provide an estimate of the duration of a specific threshold exceedance. The magnitude of the estimate is related to the size and severity of the event, causing the deviation from predicted water levels, thereby providing an objective basis for a decision to use or ignore the system during periods of degraded data.

The essence of this suggested new approach is that the assumption of spatial uniformity is likely to be met, even though the threshold is exceeded, if data from the recent past are used to distinguish short-duration events from long-duration events, which alter the fundamental physical setting. Alteration of the physical setting for extended periods affects the site-specific tidal amplitude and phasing relationships upon which the ARTTES calculations are based, thereby producing potentially erroneous values. The duration limit is based upon knowledge of the history of site water-level data and engineering judgment.

By using combinations of levels of threshold exceedance and duration, practical aspects of the dredging process may be taken into account. Because dredges are costly to operate, data access denial should occur only when the

potential consequences warrant. In other words, data degraded to a known degree are usually preferable to no data at all. For example, the dredging supervisor may deem it preferable to permit a relatively high level of threshold exceedance for a relatively short time; say, an estimated duration of 1 hr, rather than deny access.

Two applications were made of the Kriging and autoregressive procedures. The first was to a synthesized time series (Appendix A) of known statistical properties. The values of the variables THOLD and NPF for this application were 0.25 m and 10 time-steps, respectively. Results of this application are given in Appendix C. In general, periods of access denial for this simulation are not found until the latter part of the time series, first at time-step 1200 and again at time-step 1600. Beginning at time-step 1637, there is an extended period of access denial that continues until time-step 1683, or 47 time-steps. Examination of the input time series for this period shows residual values exceeding the threshold, often by factors of two or more. Moreover, comparison of the input and output time series shows that short periods of threshold exceedance in the input time series are smoothed out by the low-pass filter and therefore do not invoke application of the Kriging procedure.

The Kriging and autoregressive procedures also were applied to a residual time series acquired at an operating ARTTES system. This time series is typical of periods when weather conditions are unsuitable for survey vessel operation but when dredges were likely to continue operating. The threshold value and acceptable duration for this application were set at 0.2 m and 3 hr, respectively. Results of the applied Kriging procedure are shown in Figure 1. The autoregressive procedure produced almost identical results. Both periods during which user access is denied under the present system and the Kriging procedure are illustrated. Note the shorter period of user denial with the Kriging procedure when compared with the existing system. As with the synthesized time series, a low-pass filter was applied to the input time series to remove high-frequency components before applying the Kriging or autoregressive procedures.

Summary and Conclusions

Objective estimation procedures have been developed to augment the present method, which denies users access to ARTTES data during periods of poor weather conditions. The procedures, termed "kriging" and "autoregression," provide an estimate of the duration that a given residual threshold will be exceeded. This estimate can be used by project managers as a decision basis for permitting dredge operators continued access to the system during periods of possible degraded data.

A computer code is provided for determination of Kriging and autoregressive estimates of future residual values (V_n) based upon residual values at the

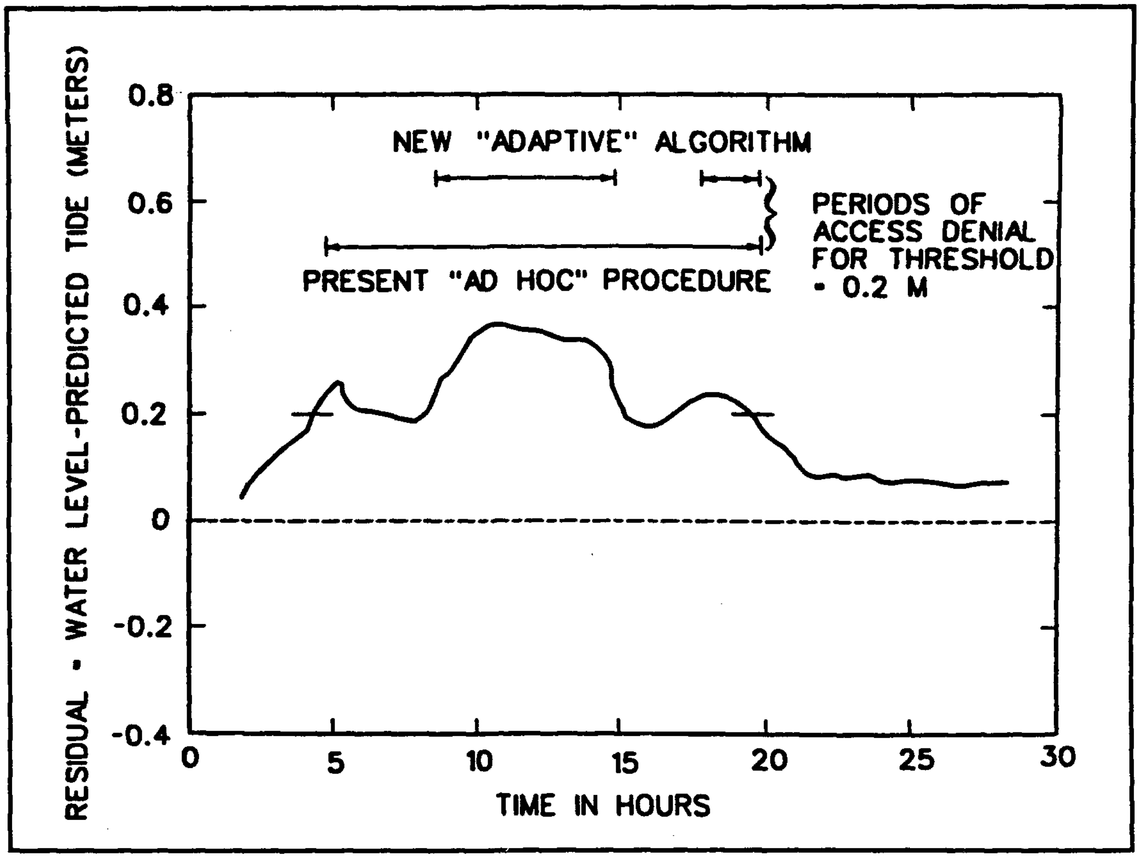

Figure 1. Comparison of ARTTES access periods for ad hoc and adaptive procedures for 0.2-m threshold

preceding time-steps. The code includes a low-pass filter with user-specified cut-off frequency (FLPASS).

Two estimation procedures were evaluated during this study; autoregression and Kriging. It was determined that although the Kriging procedure has a slightly larger rms error than the autoregression procedure, the Kriging estimate is unbiased whereas the autoregression estimate shows bias.

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Appendix A
Input Time Series, GARSHOR.DAT

Table A1

(Tens of step numbers in left column, units for steps across columns, time series values in body of table)

	0	1	2	3	4	5	6	7	8	9
0		0.07	0.12	0.14	0.15	0.14	0.09	0.09	0.08	0.10
10	0.15	0.24	0.31	0.33	0.31	0.23	0.16	0.09	0.10	0.10
20	0.08	0.07	0.04	0.05	0.05	0.07	0.09	0.08	0.08	0.09
30	0.02	0.01	0.01	0.03	0.08	0.18	0.29	0.31	0.27	0.18
40	0.11	0.08	0.09	0.06	0.02	-0.04	-0.23	-0.40	-0.52	-0.53
50	-0.43	-0.37	-0.28	-0.14	-0.12	-0.11	-0.07	0.00	-0.01	-0.01
60	-0.03	0.00	0.06	0.08	0.09	0.07	0.07	0.05	-0.02	-0.07
70	-0.06	-0.01	0.05	0.10	0.14	0.16	0.17	0.14	0.12	0.12
80	0.16	0.22	0.24	0.23	0.22	0.24	0.26	0.26	0.25	0.22
90	0.19	0.17	0.16	0.17	0.16	0.17	0.15	0.12	0.10	0.08
100	0.12	0.16	0.21	0.28	0.28	0.23	0.15	0.13	0.06	0.04
110	0.03	0.00	-0.01	-0.06	-0.08	-0.10	-0.10	-0.04	-0.01	-0.05
120	-0.10	-0.09	-0.10	-0.09	-0.08	-0.04	-0.03	-0.02	0.00	0.04
130	0.08	0.11	0.11	0.06	-0.01	-0.04	-0.06	-0.01	0.06	0.10
140	0.07	0.08	0.08	0.04	-0.02	0.00	0.07	0.10	0.11	0.13
150	0.09	0.08	0.15	0.20	0.25	0.28	0.23	0.12	0.03	-0.02
160	-0.04	-0.02	0.03	0.05	0.04	0.04	0.05	0.03	-0.01	-0.04
170	-0.03	-0.01	-0.02	-0.02	-0.01	0.01	0.07	0.16	0.22	0.26
180	0.23	0.15	0.07	-0.02	-0.05	-0.04	0.02	0.09	0.10	0.16
190	0.17	0.18	0.17	0.18	0.17	0.20	0.19	0.17	0.11	0.12
200	0.12	0.18	0.33	0.48	0.59	0.58	0.45	0.25	0.12	0.10
210	0.10	0.10	0.08	0.11	0.13	0.18	0.23	0.22	0.19	0.16
220	0.09	0.04	0.03	0.08	0.11	0.17	0.22	0.31	0.35	0.34
230	0.29	0.19	0.07	0.01	0.01	0.03	0.06	0.10	0.14	0.20
240	0.23	0.28	0.27	0.25	0.21	0.17	0.12	0.07	0.07	0.10
250	0.18	0.24	0.26	0.26	0.21	0.09	-0.05	-0.10	-0.10	-0.03
260	-0.02	-0.04	-0.05	-0.03	-0.03	-0.03	-0.01	0.01	-0.01	-0.07
270	-0.11	-0.13	-0.15	-0.15	-0.14	-0.12	-0.11	-0.08	-0.07	-0.10
280	-0.14	-0.19	-0.18	-0.15	-0.28	-0.24	-0.16	-0.04	0.08	0.10
290	0.12	0.16	0.19	0.18	0.12	0.09	0.13	0.13	0.12	0.12
300	0.13	0.19	0.24	0.26	0.24	0.21	0.20	0.16	0.12	0.09
310	0.06	0.05	0.05	0.07	0.05	0.07	0.08	0.08	0.03	0.00
320	0.00	0.04	0.04	0.03	0.02	0.02	-0.02	-0.07	-0.09	-0.08
330	-0.05	-0.06	-0.11	-0.15	-0.17	-0.16	-0.17	-0.18	-0.18	-0.17
340	-0.11	-0.09	-0.11	-0.11	-0.11	-0.07	-0.08	-0.11	-0.15	-0.19
350	-0.21	-0.23	-0.22	-0.17	-0.12	-0.09	-0.10	-0.13	-0.16	-0.13
360	-0.11	-0.07	-0.06	-0.05	-0.05	-0.04	-0.04	0.01	0.09	0.17
370	0.20	0.21	0.19	0.14	0.03	-0.04	-0.08	-0.05	0.02	0.07
380	0.07	0.05	0.02	-0.02	-0.05	-0.07	-0.09	-0.10	-0.10	-0.09
390	-0.08	-0.04	0.01	0.06	0.11	0.12	0.11	0.09	0.03	-0.03

(Continued)

Table A1 (Continued)

	0	1	2	3	4	5	6	7	8	9
400	-0.10	-0.13	-0.10	-0.06	-0.06	-0.09	-0.13	-0.17	-0.20	-0.19
410	-0.17	-0.17	-0.21	-0.28	-0.28	-0.23	-0.16	-0.07	0.03	0.11
420	0.10	0.04	-0.04	-0.11	-0.19	-0.19	-0.17	-0.11	-0.09	-0.10
430	-0.14	-0.16	-0.17	-0.19	-0.17	-0.13	-0.11	-0.14	-0.18	-0.17
440	-0.12	-0.05	-0.01	0.02	0.02	0.01	-0.04	-0.10	-0.16	-0.19
450	-0.18	-0.17	-0.17	-0.15	-0.14	-0.17	-0.20	-0.18	-0.19	-0.20
460	-0.18	-0.18	-0.18	-0.10	0.00	0.10	0.16	0.22	0.26	0.24
470	0.20	0.15	0.06	0.04	0.02	0.02	0.09	0.19	0.24	0.28
480	0.25	0.19	0.11	0.07	0.01	0.06	0.02	0.04	0.14	0.25
490	0.38	0.47	0.48	0.46	0.37	0.22	0.13	0.04	0.04	0.09
500	0.09	0.15	0.28	0.42	0.50	0.44	0.37	0.26	0.12	0.03
510	-0.03	0.00	0.18	0.34	0.43	0.52	0.59	0.58	0.50	0.38
520	0.22	0.16	0.16	0.19	0.23	0.24	0.28	0.31	0.31	0.26
530	0.22	0.22	0.19	0.17	0.17	0.21	0.26	0.33	0.36	0.37
540	0.34	0.31	0.23	0.15	0.11	0.15	0.16	0.17	0.16	0.15
550	0.14	0.16	0.14	0.11	0.05	0.02	0.03	0.03	0.03	0.09
560	0.14	0.18	0.20	0.20	0.16	0.11	0.04	-0.01	-0.04	0.00
570	0.02	0.04	0.06	0.04	0.01	0.01	-0.01	-0.03	-0.07	-0.08
580	-0.07	-0.05	-0.02	0.04	0.12	0.17	0.15	0.15	0.12	0.08
590	0.01	-0.06	-0.08	-0.05	-0.01	0.03	0.05	0.04	0.02	0.01
600	0.00	-0.03	-0.08	-0.11	-0.11	-0.12	-0.14	-0.08	0.00	0.09
610	0.10	0.08	0.08	0.06	-0.01	-0.06	-0.11	-0.10	-0.03	0.04
620	0.07	0.08	0.06	0.04	0.03	-0.02	-0.06	-0.04	0.00	0.04
630	0.07	0.11	0.21	0.31	0.35	0.37	0.37	0.34	0.27	0.20
640	0.12	0.10	0.14	0.18	0.17	0.17	0.13	0.10	0.08	0.05
650	0.03	0.01	0.00	0.00	-0.01	0.01	0.07	0.16	0.19	0.22
660	0.21	0.15	0.09	0.01	-0.04	-0.05	-0.04	-0.01	0.01	0.03
670	0.04	0.04	0.01	-0.02	-0.03	-0.05	-0.08	-0.10	-0.12	-0.10
680	-0.04	0.04	0.11	0.14	0.10	0.03	-0.06	-0.13	-0.17	-0.18
690	-0.18	-0.16	-0.12	-0.10	-0.13	-0.19	-0.22	-0.25	-0.26	-0.29
700	-0.30	-0.28	-0.27	-0.24	-0.16	-0.06	0.05	0.09	0.11	0.09
710	0.06	-0.01	-0.08	-0.10	-0.08	-0.02	0.03	0.04	0.08	0.07
720	0.04	0.00	-0.02	-0.05	-0.07	-0.12	-0.16	-0.16	-0.13	-0.04
730	0.02	0.06	0.08	0.06	0.03	-0.04	-0.10	-0.11	-0.10	-0.09
740	-0.09	-0.06	-0.02	0.02	0.01	-0.03	-0.08	-0.12	-0.15	-0.18
750	-0.17	-0.13	-0.10	-0.08	-0.07	-0.05	-0.01	0.01	0.00	-0.09
760	-0.18	-0.22	-0.22	-0.22	-0.23	-0.21	-0.14	-0.11	-0.11	-0.12
770	-0.12	-0.11	-0.12	-0.14	-0.16	-0.15	-0.12	-0.09	-0.05	-0.04
780	-0.03	-0.02	-0.05	-0.11	-0.17	-0.19	-0.20	-0.18	-0.18	-0.19
790	-0.21	-0.20	-0.21	-0.20	-0.18	-0.15	-0.14	-0.17	-0.18	-0.16
800	-0.10	-0.05	0.00	0.00	-0.04	-0.03	-0.01	-0.01	0.02	0.03
810	0.06	0.08	0.11	0.09	0.09	0.09	0.12	0.11	0.08	0.10
820	0.14	0.17	0.21	0.25	0.29	0.32	0.34	0.31	0.29	0.22
830	0.15	0.11	0.11	0.14	0.14	0.16	0.16	0.15	0.14	0.06
840	0.00	-0.02	0.01	0.04	0.09	0.15	0.19	0.22	0.27	0.31
850	0.35	0.30	0.22	0.11	0.01	-0.05	-0.04	0.01	0.09	0.14
860	0.17	0.15	0.12	0.07	-0.01	-0.06	-0.06	-0.05	-0.04	0.00
870	0.07	0.13	0.18	0.25	0.29	0.28	0.18	0.04	-0.09	-0.17
880	-0.19	-0.13	-0.05	0.00	0.02	0.06	0.07	0.03	-0.05	-0.13
890	-0.15	-0.14	-0.15	-0.14	-0.10	-0.02	0.07	0.15	0.22	0.27
900	0.23	0.15	0.04	-0.08	-0.15	-0.12	-0.03	0.04	0.04	0.07
910	0.09	0.09	0.09	0.07	0.01	-0.04	-0.10	-0.12	-0.15	-0.12
920	-0.06	0.04	0.15	0.26	0.32	0.27	0.13	-0.02	-0.10	-0.11
930	-0.08	-0.05	-0.02	-0.02	0.03	0.10	0.12	0.11	0.10	0.04

(Continued)

Table A1 (Continued)

	0	1	2	3	4	5	6	7	8	9
940	0.01	-0.03	-0.07	-0.03	0.01	0.00	-0.01	0.05	0.11	0.16
950	0.08	-0.05	-0.14	-0.14	-0.13	-0.09	-0.06	-0.02	-0.02	0.01
960	0.01	0.01	0.04	0.07	0.03	-0.03	-0.06	-0.06	-0.04	-0.03
970	-0.05	-0.03	0.02	0.04	0.00	-0.06	-0.11	-0.11	-0.11	-0.16
980	-0.17	-0.13	-0.14	-0.16	-0.19	-0.16	-0.12	-0.09	-0.05	-0.04
990	-0.07	-0.11	-0.13	-0.13	-0.13	-0.10	-0.05	-0.03	-0.01	-0.04
1000	-0.08	-0.08	-0.06	-0.09	-0.10	-0.06	-0.05	-0.09	-0.12	-0.15
1010	-0.13	-0.06	0.01	0.06	0.07	0.06	0.05	0.07	0.05	0.03
1020	0.02	0.01	-0.03	-0.08	-0.08	-0.05	-0.01	-0.03	-0.08	-0.09
1030	-0.11	-0.12	-0.16	-0.19	-0.21	-0.16	-0.13	-0.12	-0.09	-0.04
1040	0.00	-0.01	-0.03	-0.07	-0.13	-0.16	-0.21	-0.25	-0.25	-0.17
1050	-0.12	-0.11	-0.12	-0.16	-0.20	-0.24	-0.27	-0.31	-0.33	-0.32
1060	-0.28	-0.26	-0.20	-0.10	-0.01	0.03	0.05	0.01	-0.04	-0.11
1070	-0.19	-0.25	-0.24	-0.17	-0.10	-0.06	-0.06	-0.09	-0.14	-0.20
1080	-0.23	-0.26	-0.27	-0.27	-0.28	-0.27	-0.22	-0.15	-0.08	0.00
1090	0.05	0.03	0.00	-0.04	-0.11	-0.19	-0.25	-0.22	-0.13	-0.07
1100	0.02	0.11	-0.02	-0.09	-0.12	-0.12	-0.16	-0.18	-0.16	-0.10
1110	-0.03	0.11	0.28	0.41	0.48	0.44	0.37	0.24	0.15	0.04
1120	-0.02	-0.01	0.07	0.19	0.32	0.38	0.37	0.34	0.27	0.16
1130	0.08	0.04	0.05	0.08	0.14	0.23	0.37	0.48	0.54	0.53
1140	0.47	0.40	0.31	0.18	0.12	0.14	0.21	0.28	0.34	0.39
1150	0.41	0.38	0.28	0.21	0.14	0.12	0.14	0.14	0.15	0.19
1160	0.25	0.33	0.40	0.39	0.36	0.31	0.24	0.17	0.11	0.13
1170	0.17	0.21	0.24	0.26	0.26	0.25	0.19	0.13	0.09	0.11
1180	0.14	0.16	0.20	0.21	0.24	0.30	0.34	0.35	0.31	0.29
1190	0.27	0.20	0.17	0.16	0.19	0.22	0.24	0.27	0.30	0.28
1200	0.20	0.13	0.08	0.10	0.14	0.15	0.18	0.19	0.20	0.23
1210	0.25	0.23	0.17	0.09	0.00	-0.05	-0.07	-0.05	-0.02	0.03
1220	0.06	0.08	0.12	0.12	0.10	0.01	-0.04	0.02	0.02	0.04
1230	0.09	0.15	0.15	0.14	0.17	0.17	0.11	0.06	0.02	0.00
1240	-0.01	-0.03	-0.01	0.00	0.00	0.02	-0.02	-0.06	-0.10	-0.18
1250	-0.19	-0.15	-0.05	-0.04	-0.05	-0.02	0.00	0.04	0.06	0.07
1260	0.02	-0.01	-0.04	-0.11	-0.09	-0.12	-0.09	-0.10	-0.18	-0.22
1270	-0.23	-0.26	-0.35	-0.43	-0.42	-0.29	-0.08	0.04	0.12	0.20
1280	0.22	0.19	0.13	0.03	-0.10	-0.18	-0.20	-0.22	-0.12	-0.04
1290	0.07	0.11	0.02	-0.09	-0.19	-0.22	-0.30	-0.42	-0.59	-0.67
1300	-0.56	-0.37	-0.13	0.08	0.15	0.12	0.07	-0.01	-0.09	-0.17
1310	-0.20	-0.23	-0.19	-0.08	0.04	0.17	0.20	0.19	0.11	0.00
1320	-0.15	-0.29	-0.36	-0.34	-0.24	-0.11	0.04	0.23	0.39	0.40
1330	0.35	0.26	0.15	0.04	-0.05	-0.09	-0.05	0.06	0.19	0.31
1340	0.41	0.42	0.33	0.18	0.02	-0.14	-0.22	-0.24	-0.21	-0.11
1350	0.04	0.20	0.34	0.38	0.38	0.31	0.22	0.04	-0.10	-0.22
1360	-0.24	-0.18	-0.04	0.09	0.19	0.20	0.11	0.03	-0.10	-0.23
1370	-0.36	-0.43	-0.42	-0.31	-0.17	0.01	0.19	0.32	0.40	0.37
1380	0.24	0.05	-0.09	-0.20	-0.22	-0.14	0.00	0.14	0.29	0.39
1390	0.38	0.29	0.15	-0.02	-0.21	-0.33	-0.36	-0.35	-0.25	-0.15
1400	0.03	0.22	0.32	0.31	0.21	0.05	-0.13	-0.31	-0.40	-0.40
1410	-0.30	-0.16	0.05	0.22	0.28	0.27	0.17	0.03	-0.18	-0.26
1420	-0.32	-0.34	-0.34	-0.23	-0.01	0.23	0.39	0.48	0.48	0.33
1430	0.13	-0.10	-0.25	-0.26	-0.15	-0.01	0.14	0.28	0.45	0.48
1440	0.45	0.35	0.17	0.03	-0.08	-0.18	-0.25	-0.20	-0.05	0.16
1450	0.39	0.58	0.64	0.59	0.43	0.21	0.03	-0.15	-0.16	-0.08
1460	0.09	0.23	0.40	0.59	0.68	0.66	0.52	0.41	0.25	0.12
1470	0.02	-0.03	0.04	0.14	0.25	0.41	0.52	0.58	0.57	0.36
1480	0.13	-0.09	-0.24	-0.28	-0.22	-0.10	0.05	0.23	0.40	0.49
1490	0.47	0.37	0.25	0.05	-0.10	-0.22	-0.25	-0.17	-0.02	0.13

(Continued)

Table A1 (Continued)

	0	1	2	3	4	5	6	7	8	9
1500	0.31	0.44	0.45	0.35	0.17	-0.03	-0.20	-0.31	-0.33	-0.26
1510	-0.17	-0.03	0.14	0.30	0.40	0.46	0.42	0.31	0.13	-0.04
1520	-0.16	-0.20	-0.16	-0.08	0.07	0.26	0.38	0.40	0.34	0.21
1530	0.03	-0.17	-0.31	-0.37	-0.37	-0.25	-0.06	0.12	0.31	0.44
1540	0.50	0.46	0.32	0.15	0.01	-0.08	-0.14	-0.19	-0.16	-0.02
1550	0.16	0.30	0.39	0.40	0.32	0.13	-0.11	-0.33	-0.45	-0.47
1560	-0.37	-0.19	0.01	0.20	0.36	0.43	0.35	0.23	0.13	0.01
1570	-0.10	-0.17	-0.17	-0.09	0.05	0.24	0.40	0.51	0.53	0.42
1580	0.25	0.07	-0.04	-0.14	-0.19	-0.09	0.09	0.26	0.47	0.60
1590	0.62	0.56	0.49	0.43	0.32	0.26	0.13	0.08	0.12	0.20
1600	0.29	0.44	0.54	0.53	0.49	0.37	0.21	0.08	-0.04	-0.05
1610	-0.04	0.07	0.26	0.39	0.51	0.61	0.63	0.60	0.48	0.34
1620	0.19	0.10	0.13	0.17	0.26	0.42	0.56	0.64	0.68	0.63
1630	0.49	0.36	0.23	0.14	0.11	0.17	0.26	0.40	0.54	0.64
1640	0.70	0.70	0.63	0.52	0.40	0.28	0.17	0.10	0.13	0.28
1650	0.50	0.69	0.77	0.72	0.61	0.45	0.30	0.17	0.09	0.11
1660	0.18	0.28	0.42	0.56	0.67	0.70	0.67	0.55	0.39	0.26
1670	0.13	0.03	0.00	0.13	0.34	0.54	0.70	0.71	0.63	0.47
1680	0.27	0.07	-0.09	-0.13	-0.08	0.02	0.15	0.32	0.50	0.60
1690	0.57	0.46	0.29	0.14	0.01	-0.13	-0.17	-0.08	0.10	0.30
1700	0.47	0.55	0.53	0.44	0.28	0.09	-0.07	-0.10	-0.04	-0.02
1710	0.11	0.29	0.49	0.59	0.61	0.58	0.48	0.36	0.22	0.12
1720	0.05	0.05	0.11	0.20	0.36	0.48	0.57	0.55	0.44	0.23
1730	0.03	-0.12	-0.15	-0.13	-0.01	0.11	0.28	0.40	0.49	0.46
1740	0.43	0.32	0.14	-0.01	-0.12	-0.14	-0.07	0.01	0.14	0.29
1750	0.40	0.43	0.39	0.25	0.06	-0.10	-0.22	-0.28	-0.25	-0.17
1760	-0.03	0.10	0.23	0.26	0.24	0.17	0.05	-0.08	-0.20	-0.25
1770	-0.27	-0.23	-0.16	-0.07	0.05	0.16	0.18	0.15	0.04	-0.10
1780	-0.22	-0.33	-0.35	-0.31	-0.17	-0.02	0.10	0.21	0.25	0.23
1790	0.16	0.03	-0.10	-0.18	-0.21	-0.19	-0.12	-0.03	0.07	0.20
1800	0.34	0.37	0.34	0.23	0.09	-0.02	-0.06	-0.07	0.00	0.10
1810	0.21	0.30	0.37	0.41	0.37	0.27	0.14	0.04	-0.04	-0.10
1820	-0.11	-0.10	-0.03	0.05	0.16	0.24	0.27	0.24	0.14	0.02
1830	-0.08	-0.13	-0.15	-0.12	-0.04	0.04	0.13	0.23	0.24	0.23
1840	0.16	0.05	-0.05	-0.12	-0.18	-0.20	-0.18	-0.07	0.05	0.12
1850	0.17	0.22	0.20	0.10	-0.02	-0.10	-0.14	-0.18	-0.14	-0.01
1860	0.09	0.21	0.33	0.37	0.37	0.26	0.17	0.12	0.07	0.06
1870	0.07	0.08	0.15	0.25	0.34	0.40	0.45	0.43	0.36	0.26
1880	0.21	0.16	0.11	0.11	0.13	0.21	0.28	0.33	0.32	0.28
1890	0.24	0.20	0.10	0.04	-0.03	-0.07	-0.06	-0.02	0.08	0.19
1900	0.31	0.36	0.31	0.18	0.04	-0.05	-0.12	-0.17	-0.20	-0.11
1910	-0.04	0.03	0.10	0.16	0.13	0.06	-0.06	-0.17	-0.25	-0.31
1920	-0.32	-0.29	-0.14	0.03	0.23	0.37	0.35	0.27	0.21	0.15
1930	0.11	0.04	0.05	0.15	0.24	0.28	0.32	0.37	0.40	0.38
1940	0.35	0.30	0.24	0.20	0.21	0.22	0.24	0.30	0.40	0.50
1950	0.58	0.58	0.55	0.56	0.49	0.44	0.35	0.34	0.33	0.36
1960	0.42	0.47	0.53	0.51	0.45	0.45	0.31	0.18	0.09	0.03
1970	0.01	0.05	0.20	0.34	0.44	0.47	0.39	0.32	0.19	0.08
1980	-0.02	-0.03	-0.02	0.01	0.06	0.17	0.27	0.32	0.27	0.16
1990	0.02	-0.07	-0.18	-0.28	-0.32	-0.22	-0.03	0.14	0.25	0.31
2000	0.33	0.26	0.16	0.06	-0.03	-0.08	-0.10	-0.06	0.04	0.17
2010	0.30	0.37	0.34	0.23	0.11	-0.01	-0.14	-0.23	-0.27	-0.23
2020	-0.13	0.03	0.18	0.28	0.35	0.34	0.24	0.11	-0.03	-0.12

(Continued)

Table A1 (Concluded)

	0	1	2	3	4	5	6	7	8	9
2030	-0.14	-0.11	-0.03	0.12	0.27	0.39	0.41	0.31	0.16	0.00
2040	-0.15	-0.28	-0.36	-0.35	-0.25	-0.10	0.08	0.21	0.31	0.32
2050	0.26	0.13	-0.04	-0.16	-0.24	-0.25	-0.20	-0.05	0.14	0.29
2060	0.39	0.34	0.23	0.05	-0.11	-0.25	-0.38	-0.43	-0.39	-0.25
2070	-0.09	0.05	0.19	0.30	0.28	0.18	-0.02	-0.19	-0.31	-0.34
2080	-0.31	-0.20	-0.02	0.17	0.31	0.36	0.30	0.19	0.05	-0.13
2090	-0.29	-0.41	-0.45	-0.37	-0.24	-0.09	0.10	0.26	0.29	0.19
2100	0.05	-0.13	-0.28	-0.38	-0.41	-0.35	-0.17	0.04	0.21	0.34
2110	0.38	0.36	0.26	0.09	-0.13	-0.29	-0.39	-0.42	-0.38	-0.27
2120	-0.03	0.20	0.31	0.34	0.26	0.09	-0.12	-0.29	-0.42	-0.49
2130	-0.38	-0.15	0.11	0.23	0.32	0.34	0.32			

Appendix B
Prediction Computer Program

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      PROGRAM PREDICT6
C *************************************************************
C    PREDICT (WITH ERROR ESTIMATE) J0+M,J0+M+1,J0+M+2,...,J0+M+NPF-1
C       TIME-STEP-VALUES BASED ON THE VALUES AT TIME STEPS
C       (J0,J0+1,J0+2,J0+3,...,J0+M-1)
C       AND FLAG ANY SERIES VALUES MORE THAN K*SIGMA
C       FROM THEIR PREDICTORS (NORMALY, K=3)
C
C    THE TIME SERIES IS INITIALLY STORED IN FILE: TIMSER.DAT
C       AS {V(J);J=1,NJ}, WHERE J0>=JB>=1 AND (J0+M+NPF-1)<=NJ
C
C    MAKE COMPLETE SEQUENCE OF ESTIMATES FOR JB<=J0<=NJ-M-NPF+1
C       (NOTE: THE NUMBER OF TIME STEP SEQUENCES PREDICTED IS
C       NPTS=NJ-JB-M+1). THE PREDICTIONS ARE STORED IN
C       {VPK(JJ,LL);JJ=1,NPTS;LL=1,NPF} (FOR KRIGING ESTIMATES)
C       AND {VPA(JJ,LL);JJ=1,NPTS;LL=1,NPF} (FOR AUTOREGRESSIVE
C       ESTIMATES). THE CORRESPONDING ESTIMATES OF R.M.S. ERROR
C       ARE STORED IN {ERK(JJ,LL);JJ=1,NPTS;LL=1,NPF} (FOR KRIGING)
C       AND {ERA(JJ,LL);JJ=1,NPTS;LL=1,NPF} (FOR AUTOREGRESSIVE
C       RESULTS). THE ASSOCIATED TIME STEP INDICES ARE STORED IN
C       {JTS(JJ,LL);JJ=1,NPTS;LL=1,NPF}
C
C       {VPK(JJ,LL);JJ=1,NPTS;LL=1,NPF} = KRIGING PREDICTIONS,
C          USING DATA FROM TIME STEPS JJ-M TO JJ-1, AND MAKING
C          PREDICTIONS FOR TIME STEPS JJ TO JJ+NPF-1
C       {ERK(JJ,LL);JJ=1,NPTS;LL=1,NPF} = KRIGING R.M.S. ERROR
C          AS BASED ON AND FOR TIME STEPS LISTED ABOVE
C
C       {VPA(JJ,LL);JJ=1,NPTS;LL=1,NPF} = AUTOREG. PREDICTIONS,
C          USING DATA FROM TIME STEPS JJ-M TO JJ-1, AND MAKING
C          PREDICTIONS FOR TIME STEPS JJ TO JJ+NPF-1
C       {ERA(JJ,LL);JJ=1,NPTS;LL=1,NPF} = AUTOREG. R.M.S. ERROR
C          AS BASED ON AND FOR TIME STEPS LISTED ABOVE
C
C       {JTS(JJ,LL);JJ=1,NPTS;LL=1,NPF} = TIME STEPS ASSOCIATED
C          WITH VPK AND VPA ARRAYS ABOVE
C
C    IF (IFLAG.EQ.1) PREPARE ONLY KRIGED ESTIMATES
C    IF (IFLAG.EQ.2) COMPUTE ONLY AUTOREGRESSIVE ESTIMATES
C    IF (IFLAG.EQ.3) PREPARE BOTH KRIGED AND AUTOREGRESSIVE
C                 ESTIMATES
C
C    NJ = LENGTH OF TIME SERIES DATA, (IN TIME STEPS)
C    M = LENGTH OF PREVIOUS HISTORY USED IN PREDICTION OF NEXT
C          VALUE, (IN TIME STEPS)
C    MP1 = M+1
C    MO5 = AN INTEGER LARGER THAN (MP1/5)
C    JB+M = TIME STEP NUMBER FOR THE FIRST PREDICTION
C    NPTS = NUMBER OF PREDICTED TIME STEPS
C    J0 = STARTING TIME STEP IN MAKING A CURRENT PREDICTION,
C          (J0 GREATER THAN OR EQUAL TO JB AND J0 LESS THAN OR
C          EQUAL TO NJ-M.)
C    NPF = NUMBER OF TIME STEPS INTO THE FUTURE, STARTING AT J0+M,
C          FOR WHICH PREDICTIONS ARE SIMULTANEOUSLY MADE
C        = DURATION
C    THOLD = THRESHHOLD

C    IBYPASS = 0, DON'T BYPASS SECTION
C    IBYPASS = 1, BYPASS SECTION

C    MPF = M+NPF
C    FRACT = CUMULATIVE FRACTION OF TRACE WHICH IS REGARDED AS
```

```
C          NEGLIGIBLE RELATIVE TO THE SUM OF SMALLER
C          EIGENVALUES.  THAT IS, THE EIGENVALUES ARE RANKED
C          IN ASCENDING ORDER OF ABSOLUTE VALUE, AND THE
C          CUMULATIVE SUM OF ABSOLUTE VALUES, FROM SMALLEST TO
C          LARGEST IS COMPUTED.  THE SET OF EIGENVALUES WITH
C          CUMULATIVE ABSOLUTE VALUE LESS THAN FRAC*TRACE IS
C          IGNORED IN COMPUTING THE GENERALIZED INVERSES.
C   {J0+M,J0+M+NPF-1} = INTERVAL OF TIME STEPS BEING PREDICTED AT
C         STARTING STEPM, J0
C   LPREV = NUMBER OF TIME STEPS PRIOR TO J0 TO BE USED IN
C         CORRELATION ESTIMATES
C   MPLP = M+LPREV (MUST BE A PRODUCT OF POWERS OF 2, 3, AND 5).
C   DT = TIME INCREMENT
C   FLPASS = FREQUENCY (HERTZ) AT WHICH LOWER FREQUENCIES ARE
C            PASSED WITH DISTORTION LESS THAN FFRAC.
C   WIDTH = WIDTH OF TRANSITION ZONE FOR FILTER, IN HERTZ, FROM
C           FLPASS TO A HIGHER FREQUENCY WHERE THE SIGNAL IS
C           ELIMINATED WITH REMAINDER LESS THAT FFRAC.
C   FFRAC = FRACTION PERMITTED FOR FILTER DISTORTION.

C   MVMAX = MP1*(MP1+1)/2
C   NMAX2 = MP1*MP1
C
C   THE CORRELATION FUNCTION IS ESTIMATED WITH THE VALUES IN THE
C     TIME STEP INTERVAL (J0-L, J0+M-1), WHERE (J0-L)>=1 AND
C     L>=1
C
C   JEXAM = J-VALUE FOR WHICH INTERMEDIATE OUTPUT IS PRINTED, IF
C           (IDOC.NE.0)
C   IF IDOC = 0, IT DOESN'T MATTER WHAT JEXAM IS SET TO.
C
C   WRITTEN BY LEON BORGMAN, UNIVERSITY OF WYOMING
C *************************************************************


C ***********************************************************
C FFT COMMON BLOCK ***********************************************
      PARAMETER (MAXN=3000,MAXN2=2048,MAXN3=2187,MAXN5=625)
      PARAMETER (MAXNO2=1024,MAXNO3=729,MAXNO5=125)
C     ********************************************************
      DIMENSION W(MAXN),WR(MAXN2),WI(MAXN2),WRD(MAXN3),WID(MAXN3)
      DIMENSION WRT(MAXN5),WIT(MAXN5)
      DIMENSION IRB(MAXNO2),CO(MAXNO2),SI(MAXNO2)
      DIMENSION IRBD(MAXNO3),COD(MAXNO3),SID(MAXNO3)
      DIMENSION IRBT(MAXNO5),COT(MAXNO5),SIT(MAXNO5)
      DIMENSION CO1(MAXNO5),SI1(MAXNO5),CO2(MAXNO5),SI2(MAXNO5)
      DIMENSION CO3(MAXNO5),SI3(MAXNO5)
      COMMON /FFT/W,WR,WI,WRD,WID,WRT,WIT,IRB,CO,SI,IRBD,COD,SID
      COMMON /FFT/IRBT,COT,SIT,CO1,SI1,CO2,SI2,CO3,SI3
C ***********************************************************


C     ***** PARAMETERS TO BE SET BY USER ***
      PARAMETER (NJ=2136,M=12,JB=53,NPTS=2073,LPREV=52)
      PARAMETER (MP1=13,MPLP=64,IFLAG=3)
      PARAMETER (MVMAX=496,NMAX2=961,NPF=10,MPF=22,FRAC=0.01)
      PARAMETER (MO5=7,THOLD=0.25,IBYPASS=1)
      PARAMETER (DT=1.0,FLPASS=0.04,WIDTH=0.02,FFRAC=0.05)

C     ***** DIMENSION LIST ***
      DIMENSION V(NJ),VV(NJ),CORR(MPF),CORMAT(MP1,MP1)
      DIMENSION BI(MP1,MP1),CO(MP1),CK(MP1),CR(M)
      DIMENSION VPK(NPTS,NPF),ERK(NPTS,NPF),MLAG(NPTS)
```

```
      DIMENSION VPA(NPTS,NPF),ERA(NPTS,NPF),JTS(NPTS,NPF)
      DIMENSION A(MVMAX),EIVEC(NMAX2)
      DIMENSION INDEX(100)
      DIMENSION IQB(MO5),IQT(MO5),IQA(MP1),QA(MP1)
      DIMENSION TIMSER(MPLP),WRKVEC(MPLP)


C     ***** OUTPUT FILE ***
      OPEN (2,FILE='PREDICT6.OUT',STATUS='UNKNOWN')
      OPEN (4,FILE='RESULTS.OUT',STATUS='UNKNOWN')

C     ***** INPUT DATA FILE ***
      OPEN (3,FILE='garSHOR.DAT',STATUS='OLD')

C     ***** PROGRAM CONSTANTS ***
      IDOC=0
      JEXAM=5

      WRITE (4,150) THOLD
  150 FORMAT (2X,'THRESHOLD=  ',F8.2,//)


C     ***** READ INPUT DATA ***
      READ (3,111) (VV(I),I=1,JB+M-2)
  111 FORMAT (F8.2)

C      PRINT*,'INPUT DATA READ'

C   *********** START COMPUTATIONS ***
C     ***** LOOP OVER PREDICTION TIME STEPS ***
      DO 1 JJ=1,NPTS

        ISTOP1=0
        ISTOP2=0

        J0=JB+JJ-1
        JJJ=J0+M-1
        READ (3,111) VV(JJJ)

        DO 80 LTS=1,MPLP
          TIMSER(LTS)=VV(J0-LPREV+LTS-1)
          WRKVEC(LTS)=0.0
   80   CONTINUE
        CALL LOPASS(MPLP,FLPASS,WIDTH,TIMSER,WRKVEC,DT,FFRAC)


        DO 81 LTS=1,MPLP
          V(J0-LPREV+LTS-1)=TIMSER(LTS)
   81   CONTINUE

        WRITE (4,151) JJJ,V(JJJ)
  151   FORMAT (2X,'JJJ,V(JJJ)=  ',I5,F8.2)
        PRINT*,'JJJ,V=  ',JJJ,V(JJJ)


        MAXLAG=M

C       ***** IF READ VALUE IS ABOVE THRESHHOLD, ESTIMATE ***
C       *****    NPF=DURATION FUTURE VALUES BEYOND STEP    ***
C       *****    J0+M-1                                    ***
```

```
      IF (V(J0+M-1).GE.THOLD) THEN
        WRITE (4,152) V(JJJ)
 152      FORMAT (2X,'EXCEEDANCE OF THRESHOLD FOUND, VALUE= ',
     &    F8.2)
        PRINT*,'EXCEEDANCE OF THRESHOLD FOUND, VALUE= ',
     &    V(JJJ)
C         ***** COMPUTE MEANS OF SET ***
C         PRINT*,'COMPUTING MEANS'
        SUMV=0.0
        SUMJ=0.0
        DO 3 J=J0-LPREV,J0+M-1
          SUMV=SUMV+V(J)
          SUMJ=SUMJ+J
   3    CONTINUE
        AVV=SUMV/(M+LPREV)
        AVJ=SUMJ/(M+LPREV)
C         ***** COMPUTE LINEAR TREND ***
C         PRINT*,'FITTING LINEAR TREND'
        SXX=0.0
        SYY=0.0
        SXY=0.0
        DO 5 J=J0-LPREV,J0+M-1
          SXX=SXX+(J-AVJ)*(J-AVJ)
          SYY=SYY+(V(J)-AVV)*(V(J)-AVV)
          SXY=SXY+(J-AVJ)*(V(J)-AVV)
   5    CONTINUE
        BH=SXY/SXX
        AH=AVV-BH*AVJ

        IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
          WRITE (2,200) JJ,AH,BH
 200        FORMAT (/,2X,'JJ,AH,BH= ',I5,2F12.5,//)
        END IF



C         *****COMPUTE CORRELATION FUNCTION FOR LAG = 0,...,M ***
C         PRINT*,'CORRELATION FUNCTION'
        DO 2 KK=1,MPF
          K=KK-1
C           ***** LAG K COMPUTATIONS ***
          IF (K.LE.MAXLAG) THEN
            SUM=0.0
            DO 4 J=J0-LPREV+K,J0+M-1
              JF=J-K
              SUM=SUM+(V(JF)-AH-BH*JF)*(V(J)-AH-BH*J)
   4          CONTINUE
            CORR(KK)=SUM/(M+LPREV-K)
            IF (KK.EQ.1) THEN
              COV0=CORR(KK)
              CORR(KK)=1.0
            ELSE IF (KK.GT.1) THEN
              CORR(KK)=CORR(KK)/COV0
            END IF
            IF (CORR(KK).LE.0.0) THEN
              MAXLAG=K
              CORR(KK)=0.0
            END IF
          ELSE
            CORR(KK)=0.0
          END IF
```

```
      IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
        WRITE (2,205) COVO
205       FORMAT (/,2X,'COVO= ',F12.5)
        WRITE (2,201) JJ,KK,CORR(KK)
201       FORMAT (2X,'JJ,KK,CORR= ',2I5,F12.5)
      END IF
  2  CONTINUE

C      ***** LOAD UPPER LEFT MAXX BY MAXX PART OF CORMAT ***
C       PRINT*,'SET UP CORRELATION MATRIX'
     IF (MAXLAG.GT.2) THEN
       MAXX=MAXLAG
     ELSE
       MAXX=2
     END IF
     MLAG(JJ)=MAXX
     DO 6 L=1,MAXX
       DO 7 LL=1,MAXX
         LAG=IABS(L-LL)
         K=LAG+1
         CORMAT(L,LL)=CORR(K)
  7      CONTINUE

  6  CONTINUE

C      ***** IF IFLAG=1 OR 3, DO POINT KRIGING ***
     IF (IFLAG.EQ.1.OR.IFLAG.EQ.3) THEN
C         PRINT*,'PROCEED TO MAKE KRIGED ESTIMATE'

C        ***** FILL IN THE RIGHT AND BOTTOM MARGIN OF CORMAT ***
C        *****    AND THE RIGHT-HAND VECTOR                 ***
C         PRINT*,'PREPARE KRIGING MATRIX'
       DO 8 J=J0+M-MAXX,J0+M-1
         L=J-J0-M+MAXX+1
         MAXXP=MAXX+1
         CORMAT(L,MAXXP)=AH+BH*J
         CORMAT(MAXXP,L)=CORMAT(L,MAXXP)
  8      CONTINUE
       CORMAT(MAXXP,MAXXP)=0.0

       IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
         WRITE (2,202)
202        FORMAT (//,2X,'KRIGING MATRIX:')
         CALL WRMAT(CORMAT,MAXXP,MAXXP,MP1,MP1,INDEX)
         WRITE (2,'(//)')
       END IF


C        ***** COMPUTE THE GENERALIZED INVERSE OF AUGMENTED ***
C        *****   CORMAT                                   ***
       CALL EGINV2(CORMAT,BI,FRAC,MAXXP,MP1,MVMAX,NMAX2,
    &   EIVEC,A,QA,IQA,IQB,IQT,MO5)

C        ***** COMPUTE KRIGING PREDICTIONS FOR NPF FUTURE STEPS ***
C         PRINT*,'COMPUTING KRIGING COEFFICIENTS'

C        ***** SET FLAG FOR STOPPING ***
       ISTOP1=1
       DO 30 JPF=1,NPF

C          ***** CREATE CO VECTOR ***
         DO 31 L=1,MAXX
```

```
         CO(L)=CORR(MAXX-L+1+JPF)
 31     CONTINUE
       AVP=AH+BH*(J0+M+JPF-1)
       CO(MAXXP)=AVP

       IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
         WRITE (2,134)
134        FORMAT (/,2X,'CO VECTOR:')
         DO 32 L=1,MAXXP
           WRITE (2,206) L,CO(L)
206          FORMAT (2X,'L,CO= ',I5,F12.5)
 32        CONTINUE
       END IF


C       ***** MULTIPLY BI*CO TO GET KRIGING COEF. ***
       IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
         WRITE (2,136)
136        FORMAT (/,2X,'CK VECTOR:')
       END IF
       DO 9 L=1,MAXXP
         SUM=0.0
         DO 10 LL=1,MAXXP
           SUM=SUM+BI(L,LL)*CO(LL)
 10        CONTINUE
         CK(L)=SUM

         IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
           WRITE (2,204) L,CK(L)
204          FORMAT (2X,'L,CK= ',I5,F12.5)
         END IF

  9     CONTINUE

C       ***** COMPUTE ESTIMATE, VPK, AND ERROR, ERK ***
C        PRINT*,'CALCULATING KRIGED ESTIMATE'
       SUM=0.0
       SUMM=0.0
       DO 11 L=1,MAXX
         SUM=SUM+CK(L)*CO(L)*COVO
         J=J0+M-MAXX+L-1
         SUMM=SUMM+CK(L)*V(J)
 11     CONTINUE
       ERRR=COVO-CK(MAXXP)*COVO*AVP-SUM
       ERK(JJ,JPF)=SQRT(ABS(ERRR))
       JTS(JJ,JPF)=J0+M+JPF-1
       VPK(JJ,JPF)=SUMM

C       ***** IF VPK LESS THAN THOLD, SET ISTOP1=0 ***
       IF (SUMM.LT.THOLD) ISTOP1=0

 30    CONTINUE

C     ***** STOP IF ALL VPK.GE.THOLD ***
     IF (ISTOP1.EQ.1) THEN
C        STOP
       WRITE (4,153) JJJ
153      FORMAT (2X,'KRIGING SHUTDOWN STEP AT ',I6)
       PRINT*,'KRIGING SHUTDOWN STEP AT ',JJJ

     END IF

   END IF
```

```
C         ***** IF IFLAG=2 OR 3, DO AUTOREGRESSIVE ESTIMATE ***
         IF (IFLAG.EQ.2.OR.IFLAG.EQ.3) THEN
C           PRINT*,'PROCEED WITH AUTOREGRESSIVE ESTIMATES'

C          ***** COMPUTE THE GENERALIZED INVERSE OF ***
C          *****    NON-AUGMENTED CORMAT           ***
C           PRINT*,'INVERT MATRIX IN YULE-WALKER EQUATIONS'
          CALL EGINV2(CORMAT,BI,FRAC,MAXX,MP1,MVMAX,NMAX2,
     &     EIVEC,A,QA,IQA,IQB,IQT,MO5)

C          ***** SET UP RIGHT SIDE OF YULE-WALKER EQUATION ***
          DO 33 L=1,MAXX
            CO(L)=CORR(MAXX-L+2)
  33        CONTINUE
C          ***** MULTIPLY BI*CO ***
C           PRINT*,'COMPUTING AUTOREGRESSIVE COEFFICIENTS'

          IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
            WRITE (2,137)
 137          FORMAT (/,2X,'CR VECTOR:')
          END IF

          SUMCR=0.0
          DO 12 L=1,MAXX
            SUM=0.0
            DO 13 LL=1,MAXX
              SUM=SUM+BI(L,LL)*CO(LL)
  13          CONTINUE
            CR(L)=SUM
            SUMCR=SUMCR+CR(L)

            IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
              WRITE (2,208) L,CR(L)
 208            FORMAT (2X,'L,CR= ',I5,F12.5)
            END IF

  12        CONTINUE

          IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
            WRITE (2,209) SUMCR
 209          FORMAT (//,2X,'SUM AR COEF= ',F12.5,//)
          END IF



C          ***** ESTIMATE VALUE AND ERROR FOR AR EST. ***
C          ***** COMPUTE ESTIMATE, VPA, AND ERROR, ERA ***
C           PRINT*,'CALCULATING AUTOREGRESSIVE ESTIMATES'

C          ***** CHANGE CORRELATION MATRIX TO COVARIANCE MATRIX ***
          DO 50 L=1,MAXX
            DO 51 LL=1,MAXX
              CORMAT(L,LL)=CORMAT(L,LL)*COVO
  51          CONTINUE
  50        CONTINUE

          IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
            WRITE (2,212)
 212          FORMAT (//,2X,'INITIAL COV MATRIX:')
            CALL WRMAT(CORMAT,MAXX,MAXX,MP1,MP1,INDEX)
            WRITE (2,'(//)')
          END IF
```

```
C          ***** SET FLAG FOR STOPPING ***
         ISTOP2=1
         DO 40 JPF=1,NPF
           SUMV=0.0
           DO 41 L=2,MAXX
             CCC=CR(1)*CORMAT(L,1)
             CORMAT(L,MAXXP)=CCC
           CORMAT(MAXXP,L)=2.0*CCC
             SUM=0.0
             DO 42 LL=2,MAXX
               SUM=SUM+CORMAT(L,LL)*CR(LL)
 42            CONTINUE
             CORMAT(L,MAXXP)=CORMAT(L,MAXXP)+SUM
             CORMAT(MAXXP,L)=CORMAT(MAXXP,L)+SUM
 41         CONTINUE
           SUM=0.0
           DO 43 L=2,MAXX
             SUM=SUM+CR(L)*CORMAT(MAXXP,L)
 43         CONTINUE
           CORMAT(MAXXP,MAXXP)=SUM
           DO 44 L=2,MAXX
             DO 45 LL=2,MAXX
               CORMAT(L-1,LL-1)=CORMAT(L,LL)
 45            CONTINUE
 44         CONTINUE
           DO 46 L=2,MAXX
             CORMAT(L-1,MAXXP-1)=CORMAT(L,MAXXP)
             CORMAT(MAXXP-1,L-1)=CORMAT(L,MAXXP)
 46         CONTINUE
           CORMAT(MAXX,MAXX)=CORMAT(MAXXP,MAXXP)

         IF (IDOC.NE.0.AND.JJ.EQ.JEXAM) THEN
           WRITE (2,213)
 213         FORMAT (//,2X,'NEXT COV MATRIX:')
           CALL WRMAT(CORMAT,MAXX,MAXX,MP1,MP1,INDEX)
           WRITE (2,'(//)')
         END IF

           AVP=AH+BH*(J0+M+JPF-1)
           DO 47 L=1,MAXX
             JV=J0+M+JPF-MAXX+L-2
             AVV=AH+BH*JV
             IF (JV.LT.J0+M) THEN
               WW=V(JV)-AVV
             ELSE
               WW=VPA(JJ,JPF-MAXX+L-1)-AVV
             END IF
             SUMV=SUMV+CR(L)*WW
 47         CONTINUE
           VPA(JJ,JPF)=SUMV+AVP
           JTS(JJ,JPF)=J0+M+JPF-1
           ERR=CORMAT(MAXX,MAXX)
           ERA(JJ,JPF)=SQRT(ABS(ERR))

C           ***** IF VPA LESS THAN THOLD, SET ISTOP2=0 ***
           IF (VPA(JJ,JPF).LT.THOLD) ISTOP2=0
 40       CONTINUE

C         ***** STOP IF ALL VPA.GE.THOLD ***
         IF (ISTOP2.EQ.1) THEN
C            STOP
           WRITE (4,154) JJJ
```

```
 154          FORMAT (2X,'AR SHUTDOWN STEP AT ',I6)
             PRINT*,'AR SHUTDOWN STEP AT ',JJJ
           END IF

         END IF
       END IF

   1 CONTINUE

     IF (IBYPASS.EQ.0) THEN

C       ***** PRINT PREDICTIONS ***
C        PRINT*,'PRINT PREDICTIONS'
     DO   17 JJ=1,NPTS
          JJJ=JB+M+JJ-2
          WRITE (2,130) JJJ,MLAG(JJ)
 130        FORMAT (//,1X,'LAST TIME STEP USED FOR PREDICTION= ',I5,
   &        5X,'MAXLAG= ',I5,/)

C         ***** WRITE TIME SERIES PRECEEDING PREDICTIONS ***
          JEND=JB+M+JJ-2
          JSTART=JEND-MLAG(JJ)+1
          WRITE (2,131)
 131        FORMAT (/,2X,'DATA PRECEEDING FIRST PREDICTION:')
          DO 18 J=JSTART,JEND
            WRITE (2,113) J,V(J)
 113          FORMAT (2X,I5,F10.3)
  18        CONTINUE
          WRITE (2,'(1X)')

C         ***** WRITE PREDICTIONS ***
          WRITE (2,120)
 120        FORMAT (2X,'    J',9X,'V',7X,'VPK',7X,'ERK',7X,'VPA',
   &        7X,'ERA:')
          DO 48 JPF=1,NPF
            WRITE (2,112) JTS(JJ,JPF),V(JTS(JJ,JPF)),VPK(JJ,JPF),
   &          ERK(JJ,JPF),VPA(JJ,JPF),ERA(JJ,JPF)
 112          FORMAT (2X,I5,5F10.3)
  48        CONTINUE
  17     CONTINUE
     END IF

     STOP
     END

C
     SUBROUTINE EGINV2(B,BI,FRAC,N,NMAX,MVMAX,NMAX2,EIVEC,A,
    &QA,IQA,IQB,IQT,NMO5)
C *****************************************************
C   COMPUTE GENERALIZED INVERSE FOR A SQUARE, SYMMETRIC
C   MATRIX, WITH EIGENVECTOR METHODS.
C
C   B = N BY N INPUT MATRIX
C   BI = INVERSE OF B
C   FRAC = VALUE OF EIGENVALUE CUTOFF. NEGLIGIBLE EIGENVALUES
C         SUMMING TO A VALUE LESS THAN TRACE*FRAC ARE IGNORED.
C         (EIGENVALUES ARE ELIMINATED IN ASCENDING ORDER OF
C          ABSOLUTE VALUES.)
C   NMAX = INTEGER >= N, FOR DIMENSIONING
C   MVMAX = NMAX*(NMAX+1)/2
C   NMAX2 = NMAX*NMAX
```

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C    EIVEC = NMAX2-COMPONENT WORK VECTOR
C    A = MVMAX-COMPONENT WORK VECTOR
C
C    WRITTEN BY LEON BORGMAN, UNIVERSITY OF WYOMING
C ********************************************************

      DIMENSION B(NMAX,NMAX),BI(NMAX,NMAX)
      DIMENSION A(MVMAX),EIVEC(NMAX2)
      DIMENSION IQB(NMO5),IQT(NMO5),IQA(NMAX),QA(NMAX)

C     ***** SET IC=0 TO SIGNAL SUBR EIG TO COMPUTE BOTH ***
C     *****    EIGENVALUES ANDS EIGENVECTORS          ***
      IC=0
C     ***** CREATE A VECTOR FOR SUBR. EIG AND COMPUTE TRACE ***
      TRACE=0.0
      L=0
      DO 1 J=1,N
        DO 2 I=1,J
          L=L+1
          A(L)=B(I,J)
          IF (I.EQ.J) TRACE=TRACE+B(I,J)
    2   CONTINUE
    1 CONTINUE
      CUT=TRACE*FRAC

C     ***** COMPUTE EIGENVECTORS AND EIGENVALUES ***
      CALL EIG(A,EIVEC,N,IC,MVMAX,NMAX2)

C     ***** LOAD ABSOLUTE VALUE OF EIGENVALUES FOR SORT ***
      L=0
      DO 8 J=1,N
        DO 9 I=1,J
          L=L+1
          IF (I.EQ.J) THEN
            QA(I)=ABS(A(L))
            IQA(I)=I
          END IF
    9   CONTINUE
    8 CONTINUE

      CALL FQSORT(QA,IQA,N,NMO5,IQB,IQT,NI)

C     ***** ZERO BI ***
      DO 3 I=1,N
        DO 4 J=1,N
          BI(I,J)=0.0
    4   CONTINUE
    3 CONTINUE

C     ***** COMPUTE V*(L INVERSE)*(V TRANSPOSE), STORE IN BI ***
      ESUM=0.0
      DO 5 KK=1,N
C       ***** COMPUTE WHERE THE K-TH EIGENVALUE IS IN A ***
        K=IQA(KK)
        L=K*(K+1)/2
        ESUM=ESUM+QA(KK)
        IF (ESUM.GT.CUT) THEN
C         ***** IF K-TH CUM. SUM EXCEEDS FRAC*TRACE, ADD DYADIC ***
C         *****    INTO BI                           ***
          DO 6 I=1,N
C           ***** COMPUTE WHERE THE I-TH COMPONENT OF THE ***
C           *****    K-TH EIGENVECTOR IS IN EIVEC      ***
```

```
        IK=(K-1)*N+I
        DO 7 J=1,N
C          ***** COMPUTE WHERE THE J-TH COMPONENT OF THE ***
C          *****   K-TH EIGENVECTOR IS IN EIVEC        ***
           JK=(K-1)*N+J
C          ***** SUM INTO BI ***
           BI(I,J)=BI(I,J)+EIVEC(IK)*EIVEC(JK)/A(L)
  7       CONTINUE
  6     CONTINUE
      END IF
  5 CONTINUE

     RETURN
     END

C
     SUBROUTINE EIG(A,EIVEC,N,IC,MVMAX,NMAX2)
C    ***********************************************************
C     COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC
C       MATRIX
C     A = ORIGINAL MATRIX (OF ORDER N) IS DESTROYED IN COMPUTATION.
C        THE ELEMENTS OF A ARE ORDERED INTO A VECTOR OF UPPER
C        TRIANGULAR ELEMENTS AS SHOWN IN THE SEQUENCING PATTERN
C
C          1  2  4  7 11 16     THE EIGENVALUES ARE CONTAINED
C             3  5  8 12 17     IN THE POSITIONS OF THE DESTROYED
C                6  9 13 18 ETC.  MATRIX A CORRESPONDING TO THE
C                  10 14 19     MAIN DIAGONAL. THAT IS: 1,3,6,10,
C                     15 20     15,21, ETC.
C                        21
C             THE J-TH EIGENVALUE IS IN POSITION J*(J+1)/2
C
C     EIVEC = MATRIX OF EIGENVECTORS ,STORED IN A SINGLE VECTOR,
C             ONE AFTER THE OTHER, IN SAME SEQUENCE AS EIGENVALUES.
C             THE J-TH EIGENVECTOR WOULD OCCUPY THE SEQUENCE
C             OF COMPONENTS OF EIVEC, STARTING AT COMPONENT
C             (J-1)*N+1 AND ENDING AT COMPONENT J*N
C     N = ORDER OF MATRICES A AND EIVEC
C     NMAX = MAXIMUM SIZE OF N, FOR DIMENSIONING ONLY. (NOTE: NMAX
C            IS NOT REQUIRED AS INPUT, BUT DETERMINES MVMAX
C            AND NMAX2.)
C     MV = THE DIMENSION OF THE VECTOR A, =N*(N+1)/2
C     MVMAX = MAXIMUM SIZE OF MV, FOR DIMENSIONING ONLY
C          = NMAX*(NMAX+1)/2
C     NMAX2 = THE DIMENSION OF THE VECTOR EIVEC, = NMAX**2
C     IC = 0  COMPUTE EIGENVALUES AND EIGENVECTORS
C     IC = 1  COMPUTE EIGENVALUES ONLY (EIVEC NEED NOT BE
C        DIMENSIONED BUT MUST STILL APPEAR IN CALLING
C        SEQUENCE)
C
C         DIAGONALIZATION METHOD FROM 'MATHEMATICAL
C         METHODS FOR DIGITAL COMPUTERS', EDITED BY A. RALSTON AND
C         H.S. WILF, JOHN WILEY AND SONS, NEW YORK, 1962, CHAPTER 7
C
C       FOR A DOUBLE PRECISION VERSION, REMOVE THE LEADING 'C'
C         FROM THE FOLLOWING STATEMENT:
C      DOUBLE PRECISION A,EIVEC,ANORM,ANRMX,THR,X,Y,SINX,SINX2,COSX,
C     &            COSX2,SINCS,CONST
C       ALSO CHANGE: SQRT TO DSQRT, ABS TO DABS, AND CONST IN
C         FIRST STATEMENT BELOW TO 1.0D-12
C     ******************************************************
     DIMENSION A(MVMAX),EIVEC(NMAX2)
```

```
      CONST=1.0E-6
      IF (IC.NE.1) THEN
        IQ=-N
        DO 20 J=1,N
          IQ=IQ+N
        DO 20 I=1,N
          IJ=IQ+I
          EIVEC(IJ)=0.0
          IF (I.EQ.J) THEN
            EIVEC(IJ)=1.0
          END IF
20    CONTINUE
      END IF
      ANORM=0.0
      DO 35 I=1,N
      DO 35 J=I,N
        IF (I.NE.J) THEN
          IA=I+(J*J-J)/2
          ANORM=ANORM+A(IA)*A(IA)
        END IF
35 CONTINUE
      IF (ANORM.GT.0.0) THEN
        ANORM=1.414*SQRT(ANORM)
        ANRMX=ANORM*CONST/N
        IND=0
        THR=ANORM
45      THR=THR/FLOAT(N)
50      L=1
55      M=L+1
60      MQ=(M*M-M)/2
        LQ=(L*L-L)/2
        LM=L+MQ
        IF (ABS(A(LM)).GE.THR) THEN
          IND=1
          LL=L+LQ
          MM=M+MQ
          X=0.5*(A(LL)-A(MM))
          Y=-A(LM)/ SQRT(A(LM)*A(LM)+X*X)
          IF (X.LT.0.0) THEN
            Y=-Y
          END IF
          SINX=Y/ SQRT(2.0*(1.0+( SQRT(1.0-Y*Y))))
          SINX2=SINX*SINX
          COSX= SQRT(1.0-SINX2)
          COSX2=COSX*COSX
          SINCS =SINX*COSX
          ILQ=N*(L-1)
          IMQ=N*(M-1)
          DO 125 I=1,N
            IQ=(I*I-I)/2
            IF (I.NE.L) THEN
              IF (I.NE.M) THEN
                IF (I.LT.M) THEN
                  IM=I+MQ
                ELSE IF (I.GT.M) THEN
                  IM=M+IQ
                END IF
                IF (I.LT.L) THEN
                  IL=I+LQ
                ELSE
                  IL=L+IQ
                END IF
                X=A(IL)*COSX-A(IM)*SINX
```

```
              A(IL) = X
            END IF
          END IF
          IF (IC.NE.1) THEN
            ILR = ILQ + I
            IMR = IMQ + I
            X = EIVEC(ILR)*COSX-EIVEC(IMR)*SINX
            EIVEC(IMR) = EIVEC(ILR)*SINX + EIVEC(IMR)*COSX
            EIVEC(ILR) = X
          END IF
125       CONTINUE
        X = 2.0*A(LM)*SINCS
        Y = A(LL)*COSX2 + A(MM)*SINX2-X
        X = A(LL)*SINX2 + A(MM)*COSX2 + X
        A(LM) = (A(LL)-A(MM))*SINCS + A(LM)*(COSX2-SINX2)
        A(LL) = Y
        A(MM) = X
      END IF
      IF (M.NE.N) THEN
        M = M + 1
        GO TO 60
      ELSE
        IF (L.NE.N-1) THEN
          L = L + 1
          GO TO 55
        ELSE
          IF (IND.EQ.1) THEN
            IND = 0
            GO TO 50
          ELSE
            IF (THR.GT.ANRMX) GO TO 45
          END IF
        END IF
      END IF
    END IF
    IQ = -N
    DO 185 I = 1,N
      IQ = IQ + N
      LL = I + (I*I-I)/2
      JQ = N*(I-2)
    DO 185 J = I,N
      JQ = JQ + N
      MM = J + (J*J-J)/2
      IF (A(LL).LT.A(MM)) THEN
        X = A(LL)
        A(LL) = A(MM)
        A(MM) = X
        IF (IC.NE.1) THEN
          DO 180 K = 1,N
            ILR = IQ + K
            IMR = JQ + K
            X = EIVEC(ILR)
            EIVEC(ILR) = EIVEC(IMR)
            EIVEC(IMR) = X
180         CONTINUE
        END IF
      END IF
185 CONTINUE
    RETURN
    END
C
C
    SUBROUTINE WRMAT(A,M,N,MMAX,NMAX,INDEX)
```

```
C    PRINT A MXN MATRIX WITH COLUMN AND ROW NUMBERING
C    A=MATRIX, WITH 10 COLUMNS
C
C    INDEX= N-DIMENSIONAL WORK VECTOR
C    THE MATRIX A IS DIMENSIONED IN MAIN PROGRAM
C      AS A MMAX BY NMAX MATRIX
C      (M.LE.MMAX, N.LE.NMAX)
C
C    WRITTEN BY LEON BORGMAN, UNIVERSITY OF WYOMING
C ***************************************************
      DIMENSION A(MMAX,NMAX),INDEX(NMAX)
    1 FORMAT (1X,I13,9I12)
    2 FORMAT (1X,I5,10F12.5)
    9 FORMAT (1X)
      NR=N+10
      ICB=-9
      DO 3 K=1,N
        INDEX(K)=K
    3 CONTINUE
    4 NR=NR-10
      ICB=ICB+10
      IF (NR.LE.10) THEN
        ICE=ICB+NR-1
      ELSE
        ICE=ICB+9
      END IF
      WRITE (2,9)
      WRITE (2,1) (INDEX(K),K=ICB,ICE)
      DO 8 IR=1,M
        WRITE (2,2) IR,(A(IR,IC),IC=ICB,ICE)
    8 CONTINUE
      IF (NR.GT.10) GO TO 4
      RETURN
      END
C
      SUBROUTINE FQSORT(A,IA,N,NO5,IB,IT,NI)
C*****************************************************
C    SUBROUTINE SORTS THE COMPONENTS OF THE REAL VECTOR A(J),
C       1.LE.J.LE.N, IN ORDER OF INCREASING SIZE.
C
C    A(N) IS THE VECTOR TO BE SORTED, IA IS A VECTOR OF INTEGERS
C       CO-SORTED WITH A(N).
C    B(5) IS A WORK ARRAY.
C    IB(N/5) AND IT(N/5) ARE INTEGER WORK VECTORS OF DIMENSION
C       AT LEAST N/5
C    NI IS A WORK INTEGER
C
C    PROGRAM WRITTEN BY L. E. BORGMAN, LARAMIE, WYOMING
C
C    THE SUBROUTINE USES SUBSIDIARY SUBROUTINES: FMID, FRANK,
C      FSWAP, AND FSORT.
C*****************************************************
      DIMENSION IB(NO5),IT(NO5),IA(N),A(N),B(5)
      IB(1)=1
      IT(1)=N
      NI=1
   10 IF(NI.EQ.0) RETURN
      CALL FSORT(N,NO5,A,IA,B,IB,IT,NI)
      GO TO 10
      END
C
      SUBROUTINE FSWAP(A,B,IA,IB)
C *****************************************************
```

```
C ************************************************************
      T=A
      A=B
      B=T
      I=IA
      IA=IB
      IB=I
      RETURN
      END
C
      SUBROUTINE FRANK(N,NS,NT,A,IA)
C ************************************************************
C      SUBSIDIARY SUBR TO SUBR FQSORT
C ************************************************************
      DIMENSION IA(N),A(N)
      NTOT=NT-NS+1
      IF (NTOT.EQ.1) RETURN
      NTM=NT-1
      DO 10 I=NS,NTM
      IP=I+1
      DO 10 J=IP,NT
      IF (A(I).GT.A(J)) CALL FSWAP(A(I),A(J),IA(I),IA(J))
   10 CONTINUE
      RETURN
      END
C
      SUBROUTINE FMID(N,NS,NT,A,B,IA)
C ************************************************************
C      SUBSIDIARY SUBR TO FQSORT
C ************************************************************
      DIMENSION A(N),B(5),IA(N)
      I1=NS
      I5=NT
      I2=I1+(NT-NS)/4
      I3=I1+(NT-NS)/2
      I4=I1+(3*(NT-NS))/4
      B(1)=A(I1)
      B(2)=A(I2)
      B(3)=A(I3)
      B(4)=A(I4)
      B(5)=A(I5)
      DO 1 J=1,4
      JP=J+1
      DO 1 JJ=JP,5
      IF (B(J).LE.B(JJ)) GO TO 1
      Z=B(JJ)
      B(JJ)=B(J)
      B(J)=Z
    1 CONTINUE
      AA=A(I1)
      II=IA(I1)
      BB=B(3)
      IF(BB.EQ.A(I2)) CALL FSWAP(AA,A(I2),II,IA(I2))
      IF(BB.EQ.A(I3)) CALL FSWAP(AA,A(I3),II,IA(I3))
      IF(BB.EQ.A(I4)) CALL FSWAP(AA,A(I4),II,IA(I4))
      IF(BB.EQ.A(I5)) CALL FSWAP(AA,A(I5),II,IA(I5))
      A(I1)=AA
      IA(I1)=II
      RETURN
      END
C
      SUBROUTINE FSORT(N,NO5,A,IA,B,IB,IT,NI)
C ************************************************************
```

```
C ************************************************************
      DIMENSION IB(NO5),IT(NO5),IA(N),A(N),B(5)
      NS=IB(1)
      NT=IT(1)
      NTOT=NT-NS+1
      IF(NTOT.GE.10) GO TO 10
      CALL FRANK(N,NS,NT,A,IA)
      NI=NI-1
      IF(NI.GE.1) GO TO 17
      IB(1)=0
      IT(1)=0
      RETURN
   17 DO 18 JK=1,NI
      JP=JK+1
      IB(JK)=IB(JP)
      IT(JK)=IT(JP)
   18 CONTINUE
      IB(JP)=0
      IT(JP)=0
      RETURN
   10 CONTINUE
      CALL FMID(N,NS,NT,A,B,IA)
      JS=NS
      JE=NT
      JM=NS
      IGO=1
   11 CONTINUE
      IF(IGO) 13,12,12
   12 CONTINUE
      IF(A(JE).GE.A(JM).AND.JE.GT.JM) JE=JE-1
      IF(A(JE).GE.A(JM).AND.JE.GT.JM) GO TO 12
      IF(JE.LE.JM) GO TO 14
      CALL FSWAP(A(JE),A(JM),IA(JE),IA(JM))
      JM=JE
   14 IGO=-1
      GO TO 15
   13 CONTINUE
      IF(A(JS).LE.A(JM).AND.JS.LT.JM) JS=JS+1
      IF(A(JS).LE.A(JM).AND.JS.LT.JM) GO TO 13
      IF (JS.GE.JM) GO TO 16
      CALL FSWAP(A(JS),A(JM),IA(JS),IA(JM))
      JM=JS
   16 IGO=1
   15 IF(JE.GT.JS) GO TO 11
      NI=NI-1
      IBOT=JM-NS
      IUP=NT-JM
      IF(IBOT.EQ.0.OR.IUP.EQ.0) GO TO 19
      DO 20 JK=1,NI
      J=NI-JK+2
      JJ=J+1
      IB(JJ)=IB(J)
      IT(JJ)=IT(J)
   20 CONTINUE
      IB(1)=NS
      IT(1)=JM-1
      IB(2)=JM+1
      IT(2)=NT
      NI=NI+2
      RETURN
   19 NI=NI+1
      IF(IUP.EQ.0) GO TO 21
      IB(1)=JM+1
```

```
      RETURN
   21 IB(1)=NS
      IT(1)=JM-1
      RETURN
      END
C

      SUBROUTINE LOPASS(LPSTEP,FLPASS,WIDTH,TIMSER,WRKVEC,DT,FFRAC)
C *******************************************************
C     LOW PASS FILTER {TIMSER(J);J=1,LPSTEP} TO PASS CONTENT AT
C       FREQUENCIES LESS THAN FLPASS AND ELIMINATE SIGNAL AT
C       FREQUENCIES GREATER THAN FLPASS+WIDTH
C
C     LPSTEP = NUMBER OF TIME STEPS, ASSUMED TO BE FACTORABLE
C              INTO A PRODUCT OF POWERS OF 2, 3, AND 5.
C     DT = TIME INCREMENT
C     FLPASS = FREQUENCY (HERTZ) AT WHICH LOWER FREQUENCIES ARE
C              PASSED WITH DISTORTION LESS THAN FFRAC.
C     WIDTH = WIDTH OF TRANSITION ZONE FOR FILTER, IN HERTZ, FROM
C             FLPASS TO A HIGHER FREQUENCY WHERE THE SIGNAL IS
C             ELIMINATED WITH REMAINDER LESS THAT FFRAC.
C
C     SUBROUTINE USES A GAUSSIAN SHAPE
C
C     WRITTEN BY LEON BORGMAN, UNIVERSITY OF WYOMING
C ***************************************************************
      DIMENSION TIMSER(LPSTEP),WRKVEC(LPSTEP)

C      *** COMPUTE PRELIMINARY CONSTANTS ***
      DF=1.0/(LPSTEP*DT)
      FMP=FLPASS+WIDTH/2.0
      XLO2=LPSTEP/2.0
      U=1.0-FFRAC
      CALL RNORM(U,CUTOFF)
      SIG=WIDTH/(2.0*CUTOFF)
C      *** FFT THE TIME SERIES ***
      CALL FT235(LPSTEP,-1.0,0,TIMSER,WRKVEC)

C      *** FILTER THE FREQUENCY SEQUENCE ***
      DO 1 J=1,LPSTEP
        JM=J-1
        XJM=JM
        IF (XJM.LT.XLO2) THEN
C           *** NOTE: F=ABSOLUTE VALUE OF FREQUENCY ***
          F=JM*DF
        ELSE
          F=(LPSTEP-JM)*DF
        END IF
        IF (F.LE.FLPASS) THEN
          WGT=1.0
        ELSE IF (F.GT.FLPASS+WIDTH) THEN
          WGT=0.0
        ELSE
          Z=(F-FMP)/SIG
          CALL DFNORM (Z,WGT)
        END IF
        TIMSER(J)=WGT*TIMSER(J)
        WRKVEC(J)=WGT*WRKVEC(J)
    1 CONTINUE

C      *** INVERSE FFT THE FREQUENCY SEQUENCE ***
      CALL FT235(LPSTEP,1.0,0,TIMSER,WRKVEC)
```

```
      DO 2 J=1,LPSTEP
        TIMSER(J)=TIMSER(J)/LPSTEP
    2 CONTINUE

C      ***** TIMSER(J) NOW CONTAINS THE FILTERED RESULT ***
      RETURN
      END

C
      SUBROUTINE FT235(N,SGN,LIST,XR,XI)
      DIMENSION XR(1),XI(1)
C ***********************************************************
C     (XR(J),J=1,N)=REAL PART OF SEQUENCE TO BE FOURIER TRANSFORMED.
C     (XI(J),J=1,N)=IMAG. PART OF SEQUENCE TO BE FOURIER TRANSFORMED.
C     THE DISCRETE FOURIER TRANSFORM IS OF THE FORM:
C       A(M)=SUM FOR J=1 TO N OF
C            (XR+I*XI)*EXP(I*2*PI*SGN*(M-1)*(J-1)/N),
C               FOR 1.LE.M.LE.N
C     THUS, SGN DEFINES THE SIGN OF THE EXPONENT, AND IS EITHER 1.0
C          OR -1.0.
C     A TABLE OF TRIGONOMETRIC AND BIT REVERSAL FUNCTIONS ARE
C          USED IN THE FFT.  IF LIST=0, THE TABLE IS COMPUTED.  IF
C          LIST=1, THE TABLE IS NOT COMPUTED AND IS PRESUMED TO BE
C          PRESENT FROM EARLIER CALLS TO THE SUBROUTINE.
C     CONSEQUENTLY, LIST=0 ON FIRST CALL TO SUBR.
C               LIST=1 FOR SUBSEQUENT CALLS AT SAME N VALUE.
C
C     PARAMETER VALUES:
C       MAXN=LARGEST VALUE TO BE USED FOR N (LENGTH OF SEQUENCE)
C       MAXN2=LARGEST POWER OF 2 LESS THAN OR EQUAL TO MAXN
C       MAXN3=LARGEST POWER OF 3 LESS THAN OR EQUAL TO MAXN
C       MAXN5=LARGEST POWER OF 5 LESS THAN OR EQUAL TO MAXN
C       MAXNO2=MAXN2/2
C       MAXNO3=MAXN3/3
C       MAXNO5=MAXN5/5
C
C COMMON BLOCK ***********************************************
      PARAMETER (MAXN=3000,MAXN2=2048,MAXN3=2187,MAXN5=625)
      PARAMETER (MAXNO2=1024,MAXNO3=729,MAXNO5=125)
C ***********************************************************
      DIMENSION W(MAXN),WR(MAXN2),WI(MAXN2),WRD(MAXN3),WID(MAXN3)
      DIMENSION WRT(MAXN5),WIT(MAXN5)
      DIMENSION IRB(MAXNO2),CO(MAXNO2),SI(MAXNO2)
      DIMENSION IRBD(MAXNO3),COD(MAXNO3),SID(MAXNO3)
      DIMENSION IRBT(MAXNO5),COT(MAXNO5),SIT(MAXNO5)
      DIMENSION CO1(MAXNO5),SI1(MAXNO5),CO2(MAXNO5),SI2(MAXNO5)
      DIMENSION CO3(MAXNO5),SI3(MAXNO5)
      COMMON /FFT/W,WR,WI,WRD,WID,WRT,WIT,IRB,CO,SI,IRBD,COD,SID
      COMMON /FFT/IRBT,COT,SIT,CO1,SI1,CO2,SI2,CO3,SI3
C ***********************************************************
C      PARAMETER (MAXN=5000,MAXN2=4096,MAXN3=2187,MAXN5=3125)
C      PARAMETER (MAXNO2=2048,MAXNO3=729,MAXNO5=625)
C NOTE: THE ABOVE COMMON BLOCK PARAMETERS WOULD ALLOW THE USE OF
C       ANY N.LE.5000 OF THE FORM N=(2**K)*(3**L)*(5**M) FOR
C       INTEGER K, L, AND M WITHOUT CHANGING DIMENSIONS.
C THE CHOICES OF N WHICH MATCH THESE REQUIREMENTS ARE:
C       2     3     4     5     6     8     9    10
C      12    15    16    18    20    24    25    27
C      30    32    36    40    45    48    50    54
C      60    64    72    75    80    81    90    96
C     100   108   120   125   128   135   144   150
C     160   162   180   192   200   216   225   240
C     243   250   256   270   288   300   320   324
```

```
C    486   500   512   540   576   600   625   640
C    648   675   720   729   750   768   800   810
C    864   900   960   972  1000  1024  1080  1125
C   1152  1200  1215  1250  1280  1296  1350  1440
C   1458  1500  1536  1600  1620  1728  1800  1875
C   1920  1944  2000  2025  2048  2160  2187  2250
C   2304  2400  2430  2500  2560  2592  2700  2880
C   2916  3000  3072  3125  3200  3240  3375  3456
C   3600  3645  3750  3840  3888  4000  4050  4096
C   4320  4374  4500  4608  4800  4860  5000
C     HOWEVER, FOR A FIXED N, LESS COMPUTER STORAGE CAN BE
C     REQUIRED IF THE FOLLOWING "EXACT" DIMENSIONING IS
C     INTRODUCED:
C       N2=2**K
C       N3=3**L
C       N5=5**M
C       NO2=N2/2
C       NO3=N3/3
C       NO5=N5/5
C     DIMENSION W(N),WR(N2),WI(N2),WRD(N3),WID(N3),WRT(N5),
C        WIT(N5),IRB(NO2),CO(NO2),SI(NO2),IRBD(NO3),COD(NO3),
C        SID(NO3),IRBT(NO5),COT(NO5),SIT(NO5),CO1(NO5),
C        SI1(NO5),CO2(NO5),SI2(NO5),CO3(NO5),SI3(NO5)
C     THE MAXIMAL DIMENSIONING WAS TAKEN FROM THE LARGEST
C     POWER OF 2, OF 3, AND OF 5 WHICH IS LESS THAN 5000.
C
C     PROGRAMS WRITTEN BY A. YFANTIS, UNIV. NEVADA, LAS VEGAS,
C        AND LEON BORGMAN, UNIV. WYOMING, LARAMIE, WYO.
C
C ******************************************************
      NN=N
      K=0
    1 ND=NN-(NN/2)*2
        IF (ND.NE.0) GO TO 2
        K=K+1
        NN=NN/2
        IF (NN.EQ.1) GO TO 2
        GO TO 1
    2 L=0
    3 ND=NN-(NN/3)*3
        IF (ND.NE.0) GO TO 4
        L=L+1
        NN=NN/3
        IF (NN.EQ.1) GO TO 4
        GO TO 3
    4 M=0
    5 ND=NN-(NN/5)*5
        IF (ND.NE.0) GO TO 6
        M=M+1
        NN=NN/5
        IF (NN.EQ.1) GO TO 6
        GO TO 5
    6 CONTINUE
      IF (NN.NE.1) THEN
        WRITE (2,100)
  100   FORMAT ('WARNING: THE ASSUMPTIONS OF THE SUBR. ARE VIOLATED'/
     &'         N IS NOT EQUAL TO (2**K)*(3**L)*(5**M)')
        RETURN
      ELSE IF (K.NE.0.AND.L.EQ.0.AND.M.EQ.0) THEN
        NO2=N/2
        CALL FFT2(K,N,NO2,SGN,LIST,XR,XI,IRB,CO,SI)
      ELSE IF (K.EQ.0.AND.L.NE.0.AND.M.EQ.0) THEN
        NO3=N/3
```

```
      ELSE IF (K.EQ.0.AND.L.EQ.0.AND.M.NE.0) THEN
        NO5=N/5
        CALL FFT5(M,N,NO5,SGN,LIST,XR,XI,IRBT,COT,SIT,CO1,SI1,
     &  CO2,SI2,CO3,SI3)
      ELSE IF (K.NE.0.AND.L.NE.0.AND.M.EQ.0) THEN
        N2=2**K
        N3=3**L
        NO2=N2/2
        NO3=N3/3
        CALL FFT23(K,L,N,N2,N3,NO2,NO3,SGN,LIST,XR,XI,IRB,IRBD,
     &  CO,SI,COD,SID,WR,WI,WRD,WID,W)
      ELSE IF (K.NE.0.AND.L.EQ.0.AND.M.NE.0) THEN
        N2=2**K
        N5=5**M
        NO2=N2/2
        NO5=N5/5
        CALL FFT25(K,M,N,N2,N5,NO2,NO5,SGN,LIST,XR,XI,IRB,IRBT,
     &  CO,SI,COT,SIT,WR,WI,WRT,WIT,W,CO1,SI1,CO2,SI2,CO3,SI3)
      ELSE IF (K.EQ.0.AND.L.NE.0.AND.M.NE.0) THEN
        N3=3**L
        N5=5**M
        NO3=N3/3
        NO5=N5/5
        CALL FFT35(L,M,N,N3,N5,NO3,NO5,SGN,LIST,XR,XI,IRBD,IRBT,
     &  COD,SID,COT,SIT,WRD,WID,WRT,WIT,W,CO1,SI1,CO2,SI2,CO3,SI3)
      ELSE
        N2=2**K
        N3=3**L
        N5=5**M
        NO2=N2/2
        NO3=N3/3
        NO5=N5/5
        CALL FFT235(K,L,M,N,N2,N3,N5,NO2,NO3,NO5,SGN,LIST,XR,XI,
     &  IRB,IRBD,IRBT,CO,SI,COD,SID,COT,SIT,WR,WI,WRD,WID,WRT,WIT,
     &  CO1,SI1,CO2,SI2,CO3,SI3,W)
      END IF
      RETURN
      END
C
      SUBROUTINE FFT2(K,N,NO2,SGN,LIST,XR,XI,IRB,CO,SI)
C ***********************************************************
C     N=LENGTH OF DATA SERIES BEING TRANSFORMED.
C     K=LOG2(N).
C     NO2=N/2
C     SGN= +1 OR -1 ACCORDING AS THE EXPONENT IN THE FFT TRANSFORMATION
C         IS POSITIVE OR NEGATIVE.
C     IF LIST=0, THE REVERSED-BIT LIST AND THE ASSOCIATED SINES AND
C     COSINES ARE COMPUTED.  IF LIST=1, THE LIST IS NOT COMPUTED.
C     INSTEAD, THE LIST COMPUTED ON THE PREVIOUS CALL IS USED.
C     XR=REAL PART OF THE INPUT DATA SERIES. INITIALLY AND EQUALS REAL
C       PART OF OUTPUT ON RETURN FROM THE SUBR.
C     XI=IMAG. PART OF DATA SERIES ON INPUT AND IS IMAG. PART OF OUTPUT
C       ON RETURN FROM THE SUBR.
C     IRB=NO2-DIMENSIONAL VECTOR OF REVERSED BITS ON OUTPUT.
C     CO=NO2-DIMENSIONAL VECTOR OF COSINES ON OUTPUT.
C     SI=NO2-DIMENSIONAL VECTOR OF SINES ON OUTPUT.
C ***********************************************************
      DIMENSION IRB(1),CO(1),SI(1),XR(1),XI(1)
      IF (LIST.EQ.1) GO TO 3
C     THE REVERSED-BIT LIST AND TRIG. FUNCTIONS ARE GENERATED BY THE
C     PROCEDURE GIVED IN TABLE 7-VI.
      IRB(1)=0
      DO 1 J=1,K
```

```
      DO 1 I=1,ID
      IRB(I)=IRB(I)*2
      IF (J.LT.K) IRB(I+ID)=IRB(I)+1
    1 CONTINUE
      FN=N
      W=6.2831853/FN
      DO 2 I=1,ID
      FIR=IRB(I)/2
      A=FIR*W
      CO(I)=COS(A)
      SI(I)=SIN(A)
    2 CONTINUE
    3 CONTINUE
C     FOR EACH COLUMN OF THE FFT (NC=COLUMN NUMBER), CALCULATE
C     NB=NUMBER OF BLOCKS IN THE COLUMN, LB=LENGTH OF BLOCKS, LB2=HALF
C     OF BLOCK LENGTH.
      DO 4 NC=1,K
      NB=2**(NC-1)
      LB=N/NB
      LB2=LB/2
C     THE BLOCKS OF THE COLUMN ARE LOOPED OVER.  IS=SEQUENCE NUMBER
C     AT THE START OF THE BLOCK, IFF=SEQUENCE NUMBER HALF WAY THROUGH
C     BLOCK.
      DO 4 IB=1,NB
      C=CO(IB)
      S=SGN*SI(IB)
      IS=(IB-1)*LB+1
      IFF=(IB-1)*LB+LB2
C     THE VALUES IN THE NEXT COLUMN ARE COMPUTED USING THE TRIG.
C     FUNCTIONS FROM THE PRE-GENERATED LIST.
      DO 4 I=IS,IFF
      I2=I+LB2
      QR=XR(I2)*C-XI(I2)*S
      QI=XR(I2)*S+XI(I2)*C
      XR(I2)=XR(I)-QR
      XI(I2)=XI(I)-QI
      XR(I)=XR(I)+QR
      XI(I)=XI(I)+QI
    4 CONTINUE
C     THE FFT COEFFICIENTS ARE UNSCRANBLED INTO NORMAL ORDER.
      DO 6 I=1,NO2
      DO  6 L=1,2
      IR=IRB(I)+L
      II=I+(L-1)*NO2
      IF (IR.LE.II) GO TO 6
      ZR=XR(IR)
      ZI=XI(IR)
      XR(IR)=XR(II)
      XI(IR)=XI(II)
      XR(II)=ZR
      XI(II)=ZI
    6 CONTINUE
      RETURN
      END
C
      SUBROUTINE FFT3(K,N,NO3,SGN,LIST,XR,XI,IRB,CO,SI)
      DIMENSION IRB(1),CO(1),SI(1),XR(1),XI(1)
C ***************************************************
C     THE INPUT TO THE SUBROUTINE CONSISTS OF THE VARIABLES
C     N, K, NO3, SGN, LIST, XR, AND XI.  N IS THE NUMBER OF
C     MEASUREMENTS WHICH IS OF THE FORM N=3**K, NO3 IS EQUAL
C     TO N/3.  SGN IS -1 IN THE CASE OF THE FOURIER TRANSFORM,
C     AND +1 IN THE CASE OF THE INVERSE FOURIER TRANSFORM.
```

```
C    OF THE INPUT COMPLEX SEQUENCE OF NUMBERS.  AFTER THE
C    FFT3 IS PERFORMED, XR, XI, ARE THE REAL AND IMAGINARY
C    PARTS OF THE TRANSFORMED DATA.  IF LIST=0, THE REVERSE-
C    DIGIT LIST AND THE ASSOCIATED SINES AND COSINES ARE
C    COMPUTED.  IF LIST=1 THE LIST IS NOT COMPUTED.  INSTEAD
C    THE LIST COMPUTED OF THE PREVIOUS CALL IS USED.  IRB,
C    CO, AND SI, ARE WORK VECTORS.
C ****************************************************
      T=6.2831853/3.0
      T1=2.*T
      FN=N
      IF(LIST .EQ. 1) GO TO 3
      IRB(1)=0
      DO 1 J=1,K
      ID=3**(J-1)
      DO 1 I=1,ID
      IRB(I)=IRB(I)*3
      IF(J .EQ. K) GO TO 1
      IRB(I+ID)=IRB(I)+1
      IRB(I+2*ID)=IRB(I)+2
    1 CONTINUE
      DO 2 I=1,ID
      FIR=IRB(I)/FN
      A=FIR*T
      CO(I)=COS(A)
      SI(I)=SIN(A)
    2 CONTINUE
C  FOR EACH COLUMN OF THE FFT (NC=COLUMN NUMBER), NB=NUMBER
C  OF BLOCKS IN THE COLUMN, LB=LENGTH OF BLOCKS, LB3= A THIRD
C  OF BLOCK LENGTH.
    3 C1=COS(T)
      S1=SGN*SIN(T)
      C2=COS(T1)
      S2=SGN*SIN(T1)
      DO 4 NC=1,K
      NB=3**(NC-1)
      LB=N/NB
      LB3=LB/3
C  THE BLOCKS OF THE COLUMN ARE LOOPED OVER. IS=SEQUENCE NUMBER
C  AT THE START OF THE BLOCK, IFF=SEQUENCE NUMBER A THIRD WAY
C  THROUGH BLOCK
      DO 4 IB=1,NB
      C=CO(IB)
      S=SGN*SI(IB)
      IS=(IB-1)*LB+1
      IFF=(IB-1)*LB+LB3
C  THE VALUES IN THE NEXT COLUMN ARE COMPUTED USING THE TRIG.
C  FUNCTIONS FROM THE PRE-GENERATED LIST.
      DO 4 I=IS,IFF
      I2=I+LB3
      I3=I+2*LB3
      QR1=XR(I2)*C-XI(I2)*S
      QI1=XR(I2)*S+XI(I2)*C
      QR2=XR(I3)*(C*C-S*S)-2.*XI(I3)*S*C
      QI2=(C*C-S*S)*XI(I3)+2.*S*C*XR(I3)
      XR(I2)=XR(I)+C1*QR1-S1*QI1+C2*QR2-S2*QI2
      XI(I2)=XI(I)+S1*QR1+C1*QI1+S2*QR2+C2*QI2
      XR(I3)=XR(I)+C2*QR1-S2*QI1+C1*QR2-S1*QI2
      XI(I3)=XI(I)+S2*QR1+C2*QI1+S1*QR2+C1*QI2
      XR(I)=XR(I)+QR1+QR2
      XI(I)=XI(I)+QI1+QI2
    4 CONTINUE
C  THE FFT COEFFICIENTS ARE INSCRAMBLED INTO NORMAL ORDER
```

```
      DO 6 L=1,3
      IR=IRB(I)+L
      II=I+(L-1)*NO3
      IF (IR.LE.II) GO TO 6
      ZR=XR(IR)
      ZI=XI(IR)
      XR(IR)=XR(II)
      XI(IR)=XI(II)
      XR(II)=ZR
      XI(II)=ZI
    6 CONTINUE
      RETURN
      END
C
      SUBROUTINE FFT5(K,N,NO5,SGN,LIST,XR,XI,IRB,CO,SI,CO1,SI1,
     +  CO2,SI2,CO3,SI3)
      DIMENSION IRB(1),CO(1),SI(1),XR(1),XI(1),CO1(1),
     +  SI1(1),SI2(1),CO2(1),CO3(1),SI3(1)
C ***********************************************
C     THE INPUT TO THE SUBROUTINE CONSISTS OF THE VARIABLES
C     N, K, NO5, SGN, LIST, XR, XI.  N IS THE NUMBER OF MEAS-
C     UREMENTS WHICH IS OF THE FORM N=5**K, NO5 IS EQUAL TO
C     N/5.  SGN IS -1 IN THE CASE OF THE FOURIER TRANSFORM,
C     AND +1 IN THE CASE OF THE INVERSE FOURIER TRANSFORM.
C     XR AND XI ARE TRE REAL AND IMAGINARY PARTS RESPECTIVELY
C     OF THE INPUT COMPLEX SEQUENCE OF NUMBERS.  AFTER THE FFT5
C     IS PERFORMED XR, XI ARE THE REAL AND IMAGINARY PARTS OF
C     THE TRANSFORMED DATA.  IF LIST=0, THE REVERSE-DIGIT LIST
C     AND THE ASSOCIATED SINES AND COSINES ARE COMPUTED.  IF
C     LIST=1, THE LIST IS NOT COMPUTED.  INSTEAD THE LIST COM-
C     PUTED ON THE PREVIOUS CALL IS USED.  IRB, CO, SI, CO1,
C     SI1, CO2, SI2, CO3, AND SI3, ARE WORK VECTORS.
C ************************************************
      T=6.2831853/5.
      T1=2.*T
      T2=3.*T
      T3=4.*T
      FN=N
      IF(LIST .EQ. 1) GO TO 3
      IRB(1)=0
      DO 1 J=1,K
      ID=5**(J-1)
      DO 1 I=1,ID
      IRB(I)=IRB(I)*5
      IF(J .EQ. K) GO TO 1
      IRB(I+ID)=IRB(I)+1
      IRB(I+2*ID)=IRB(I)+2
      IRB(I+3*ID)=IRB(I)+3
      IRB(I+4*ID)=IRB(I)+4
    1 CONTINUE
      DO 2 I=1,ID
      FIR=IRB(I)/FN
      A=FIR*T
      CO(I)=COS(A)
      SI(I)=SIN(A)
      A1=2.*A
      CO1(I)=COS(A1)
      SI1(I)=SIN(A1)
      A2=3.*A
      CO2(I)=COS(A2)
      SI2(I)=SIN(A2)
      A3=4.*A
      CO3(I)=COS(A3)
```

```
  2  CONTINUE
C  FOR EACH COLUMN OF THE FFT (NC=COLUMN NUMBER), CALCULATE
C  NB=NUMBER OF BLOCKS IN THE COLUMN, LB=LENGTH OF BLOCKS,
C  LB5= A FIFTH OF BLOCK LENGTH
  3  S1=SGN*SIN(T)
     C1=COS(T)
     S2=SGN*SIN(T1)
     C2=COS(T1)
     S3=SGN*SIN(T2)
     C3=COS(T2)
     S4=SGN*SIN(T3)
     C4=COS(T3)
     DO 4 NC=1,K
     NB=5**(NC-1)
     LB=N/NB
     LB5=LB/5
C  THE BLOCKS OF THE COLUMN ARE LOOPED OVER, IS=SEQUENCE NUMBER
C  AT THE START OF THE BLOCK, IFF=SEQUENCE NUMBER A FIFTH WAY
C  THROUGH BLOCK.
     DO 4 IB=1,NB
     IS=(IB-1)*LB+1
     IFF=(IB-1)*LB+LB5
C  THE VALUES OF THE COLUMN ARE COMPUTED USING THE TRIG. FUNC-
C  TIONS FROM THE PRE-GENERATED LIST.
     DO 4 I=IS,IFF
     SAA=SGN*SI(IB)
     SA1=SGN*SI1(IB)
     SA2=SGN*SI2(IB)
     SA3=SGN*SI3(IB)
     I2=I+LB5
C  THE FFT COEFFICIENTS ARE UNSCRAMBLED INTO NORMAL ORDER
     I3=I+2*LB5
     I4=I+3*LB5
     I5=I+4*LB5
     QR1=XR(I2)*CO(IB)-XI(I2)*SAA
     QI1=XR(I2)*SAA+XI(I2)*CO(IB)
     QR2=XR(I3)*CO1(IB)-XI(I3)*SA1
     QI2=XR(I3)*SA1+XI(I3)*CO1(IB)
     QR3=XR(I4)*CO2(IB)-XI(I4)*SA2
     QI3=XR(I4)*SA2+XI(I4)*CO2(IB)
     QR4=XR(I5)*CO3(IB)-XI(I5)*SA3
     QI4=XR(I5)*SA3+XI(I5)*CO3(IB)
     XR(I2)=XR(I)+QR1*C1-QI1*S1+QR2*C2-QI2*S2+QR3*C3-QI3*S3+QR4*C4-QI4*+S4
     XI(I2)=XI(I)+QR1*S1+QI1*C1+QR2*S2+QI2*C2+QR3*S3+QI3*C3+QR4*S4+QI4*+C4
     XR(I3)=XR(I)+QR1*C2-QI1*S2+QR2*C4-QI2*S4+QR3*C1-QI3*S1+QR4*C3-QI4*+S3
     XI(I3)=XI(I)+QR1*S2+QI1*C2+QR2*S4+QI2*C4+QR3*S1+QI3*C1+QR4*S3+QI4*+C3
     XR(I4)=XR(I)+QR1*C3-QI1*S3+QR2*C1-QI2*S1+QR3*C4-QI3*S4+QR4*C2-QI4*+S2
     XI(I4)=XI(I)+QR1*S3+QI1*C3+QR2*S1+QI2*C1+QR3*S4+QI3*C4+QR4*S2+QI4*+C2
     XR(I5)=XR(I)+QR1*C4-QI1*S4+QR2*C3-QI2*S3+QR3*C2-QI3*S2+QR4*C1-QI4*+S1
     XI(I5)=XI(I)+QR1*S4+QI1*C4+QR2*S3+QI2*C3+QR3*S2+QI3*C2+QR4*S1+QI4*+C1
     XR(I)=XR(I)+QR1+QR2+QR3+QR4
     XI(I)=XI(I)+QI1+QI2+QI3+QI4
  4  CONTINUE
C  THE FFT COEFFICIENTS ARE UNSCRAMBLED INTO NORMAL ORDER
     DO 6 I=1,NO5
     DO 6 L=1,5
```

```
      IR=IRB(I)+L
      II=I+(L-1)*NO5
      IF (IR.LE.II) GO TO 6
      ZR=XR(IR)
      ZI=XI(IR)
      XR(IR)=XR(II)
      XI(IR)=XI(II)
      XR(II)=ZR
      XI(II)=ZI
    6 CONTINUE
      RETURN
      END
C
      SUBROUTINE FFT23(K1,K2,N,N1,N2,NO2,NO3,SGN,LIST,XR,XI,IRB,IRBD,CO,
     +SI,COD,SID,WR,WI,WRD,WID,W)
      DIMENSION XR(1),XI(1),IRB(1),IRBD(1),CO(1),SI(1),COD(1),
     +SID(1),WR(1),WI(1),WRD(1),WID(1),W(1)
C *********************************************
C     THE SUBROUTINE FFT23 JOINS THE FFT2 AND THE FFT3. WHEN
C     USED, BOTH, FFT2 AND FFT3, SUBROUTINES MUST BE PRESENT.
C     N IS THE NUMBER OF MEASUREMENTS WHICH IS OF THE FORM
C     N=(2**K1)*(3**K2). N1=2**K1 AND N2=3**K2, NO2=N1/2,
C     NO3=N2/3. SGN=-1 IF THE FOURIER TRANSFORM IS TO BE PER-
C     FORMED. IF LIST=0, THE REVERSED-DIGIT LIST AND THE AS-
C     SOCIATED SINES AND COSINES ARE COMPUTED. IF LIST=1, THE
C     LIST IS NOT COMPUTED. INSTEAD THE LIST COMPUTED ON THE
C     PREVIOUS CALL IS USED. XR IS THE REAL PART AND XI IS THE
C     IMAGINARY PART OF THE COMPLEX SEQUENCE TO BE TRANSFORMED.
C     AFTER THE FFT23 IS PERFORMED XR AND XI ARE THE REAL AND
C     IMAGINARY PART OF THE TRANSFORMED DATA. IRB, CO, AND SI
C     ARE WORK VECTORS.
C *********************************************
      TPI=6.283185307
      XN=N
      DO 30 I=1,N1
      FI=I-1
      TPNF=TPI*FI/XN
      DO 10 J=1,N2
      L=J-1
      WRD(J)=XR(N1*L+I)
      WID(J)=XI(N1*L+I)
   10 CONTINUE
      CALL FFT3(K2,N2,NO3,SGN,LIST,WRD,WID,IRBD,COD,SID)
      DO 20 J=1,N2
      L=J-1
      XJ=J-1
      THET=SGN*TPNF*XJ
      C=COS(THET)
      S=SIN(THET)
      XR(N1*L+I)=WRD(J)*C-WID(J)*S
      XI(N1*L+I)=WRD(J)*S+WID(J)*C
   20 CONTINUE
   30 CONTINUE
      DO 60 I=1,N2
      M=I-1
      DO 40 J=1,N1
      WR(J)=XR(N1*M+J)
      WI(J)=XI(N1*M+J)
   40 CONTINUE
      CALL FFT2(K1,N1,NO2,SGN,LIST,WR,WI,IRB,CO,SI)
      DO 50 J=1,N1
      XR(N1*M+J)=WR(J)
      XI(N1*M+J)=WI(J)
```

```
60 CONTINUE
   DO 70 I=1,N
   W(I)=XR(I)
70 CONTINUE
   DO 80 I=1,N2
   DO 80 J=1,N1
   XR((J-1)*N2+I)=W(N1*(I-1)+J)
80 CONTINUE
   DO 90 I=1,N
   W(I)=XI(I)
90 CONTINUE
   DO 95 I=1,N2
   DO 95 J=1,N1
   XI((J-1)*N2+I)=W(N1*(I-1)+J)
95 CONTINUE
   RETURN
   END
C
   SUBROUTINE FFT25(K1,K2,N,N1,N2,NO2,NO5,SGN,LIST,XR,XI,IRB,IRBD,
  +CO,SI,COD,SID,WR,WI,WRD,WID,W,CO1,SI1,CO2,SI2,CO3,SI3)
   DIMENSION XR(1),XI(1),IRB(1),IRBD(1)
   DIMENSION CO(1),SI(1),COD(1),SID(1),WR(1),WI(1)
   DIMENSION WRD(1),WID(1),W(1),CO1(1),SI1(1)
   DIMENSION CO2(1),CO3(1),SI2(1),SI3(1)
C ***********************************************
C   THE SUBROUTINE FFT25 JOINS THE FFT2 AND THE FFT5. HENCE
C   WHEN USED THE SUBROUTINES FFT2 AND FFT5 MUST BE PRESENT.
C   N IS THE NUMBER OF MEASUREMENTS, WHICH IS OF THE FORM
C   N=(2**K1)*(5**K2). N1=2**K1, N2=5**K2, NO2=N1/2, NO5=N2/5,
C   SGN=-1, IF THE FOURIER TRANSFORM IS TO BE PERFORMED. IF
C   LIST=0 THE REVERSED-DIGIT LIST AND THE ASSOCIATED SINES AND
C   COSINES ARE COMPUTED. IF LIST=1, THELIST IS NOT COMPUTED.
C   INSTEAD THE LIST COMPUTED ON THE PREVIOUS CALL IS USED.
C   XR, XI ARE THE REAL AND THE IMAGINARY PARTS RESPECTIVELY
C   OF THE INPUT COMPLEX SEQUENCE OF NUMBERS. AFTER THE FFT25
C   IS PERFORMED XR AND XI ARE THE REAL AND IMAGINARY PARTS
C   OF THE TRANSFORMED DATA. IRB, IRBD, CO, COD, SID, WR, WI,
C   WRD, WID, W, CO1, SI1, CO2, SI2, CO3, AND SI3 ARE WORK
C   VECTORS.
C ***********************************************
   TPI=6.283185307
   XN=N
   DO 30 I=1,N1
   FI=I-1
   TPNF=TPI*FI/XN
   DO 10 J=1,N2
   L=J-1
   WRD(J)=XR(N1*L+I)
   WID(J)=XI(N1*L+I)
10 CONTINUE
   CALL FFT5(K2,N2,NO5,SGN,LIST,WRD,WID,IRBD,COD,SID,CO1,SI1,
  +CO2,SI2,CO3,SI3)
   DO 20 J=1,N2
   L=J-1
   XJ=J-1
   THET=SGN*TPNF*XJ
   C=COS(THET)
   S=SIN(THET)
   XR(N1*L+I)=WRD(J)*C-WID(J)*S
   XI(N1*L+I)=WRD(J)*S+WID(J)*C
20 CONTINUE
30 CONTINUE
   DO 60 I=1,N2
```

```
      DO 40 J=1,N1
      WR(J)=XR(N1*M+J)
      WI(J)=XI(N1*M+J)
   40 CONTINUE
      CALL FFT2(K1,N1,NO2,SGN,LIST,WR,WI,IRB,CO,SI)
      DO 50 J=1,N1
      XR(N1*M+J)=WR(J)
      XI(N1*M+J)=WI(J)
   50 CONTINUE
   60 CONTINUE
      DO 70 I=1,N
      W(I)=XR(I)
   70 CONTINUE
      DO 80 I=1,N2
      DO 80 J=1,N1
      XR((J-1)*N2+I)=W(N1*(I-1)+J)
   80 CONTINUE
      DO 90 I=1,N
      W(I)=XI(I)
   90 CONTINUE
      DO 95 I=1,N2
      DO 95 J=1,N1
      XI((J-1)*N2+I)=W(N1*(I-1)+J)
   95 CONTINUE
      RETURN
      END
C
      SUBROUTINE FFT35(K1,K2,N,N1,N2,NO3,NO5,SGN,LIST,XR,XI,IRB,IRBD,
     +CO,SI,COD,SID,WR,WI,WRD,WID,W,CO1,SI1,CO2,SI2,CO3,SI3)
      DIMENSION XR(1),XI(1),IRB(1),IRBD(1),CO(1),SI(1),COD(1)
      DIMENSION SID(1),WR(1),WI(1),WRD(1),WID(1),W(1),CO1(1)
      DIMENSION SI1(1),CO2(1),SI2(1),CO3(1),SI3(1)
C ****************************************************
C     THE SUBROUTINE FFT35 JOINS THE FFT3 AND THE FFT5. HENCE WHEN
C     IS USED, THE SUBROUTINES FFT3 AND FFT5 MUST BE PRESENT. N IS
C     THE NUMBER OF MEASUREMENTS, WHICH IS OF THE FORM
C     N=(3**K1)*(5**K2)  N1=3**K1, N2=5**K2, NO3=N1/3, NO5=N2/5,
C     SGN=-1, IF THE FOURIER TRANSFORM IS TO BE PERFORMED, AND SGN=+1
C     IF THE INVERSE FOURIER TRANSFORM IS TO BE PERFORMED. IF LIST=0,
C     THE REVERSED-DIGIT LIST AND THE ASSOCIATED SINES AND COSINES.
C     IF LIST=1, THE LIST IS NOT COMPUTED. INSTEAD THE LIST COMPUTED
C     ON THE PREVIOUS CALL IS USED. XR, XI, ARE THE REAL AND THE
C     IMAGINARY PARTS RESPECTIVELY OF THE INPUT COMPLEX SEQUENCE OF
C     NUMBERS. AFTER THE FFT35 IS PERFORMED XR AND XI ARE THE REAL
C     AND IMAGINARY PARTS OF THE TRANSFORMED DATA. IRB, IRBD, CO, SI,
C     COD, SID, WR, WI, WRD, WID, W, CO1, SI1, CO2, SI2, CO3, AND SI3
C     ARE WORK VECTORS.
C ****************************************************
      TPI=6.283185307
      XN=N
      DO 30 I=1,N1
      FI=I-1
      TPNF=TPI*FI/XN
      DO 10 J=1,N2
      L=J-1
      WRD(J)=XR(N1*L+I)
      WID(J)=XI(N1*L+I)
   10 CONTINUE
      CALL FFT5(K2,N2,NO5,SGN,LIST,WRD,WID,IRBD,COD,SID,CO1,SI1,CO2,
     +SI2,CO3,SI3)
      DO 20 J=1,N2
      L=J-1
      XJ=J-1
```

```
      C=COS(THET)
      S=SIN(THET)
      XR(N1*L+I)=WRD(J)*C-WID(J)*S
      XI(N1*L+I)=WRD(J)*S+WID(J)*C
 20   CONTINUE
 30   CONTINUE
      DO 60 I=1,N2
      M=I-1
      DO 40 J=1,N1
      WR(J)=XR(N1*M+J)
      WI(J)=XI(N1*M+J)
 40   CONTINUE
      CALL FFT3(K1,N1,NO3,SGN,LIST,WR,WI,IRB,CO,SI)
      DO 50 J=1,N1
      XR(N1*M+J)=WR(J)
      XI(N1*M+J)=WI(J)
 50   CONTINUE
 60   CONTINUE
      DO 70 I=1,N
      W(I)=XR(I)
 70   CONTINUE
      DO 80 I=1,N2
      DO 80 J=1,N1
      XR((J-1)*N2+I)=W(N1*(I-1)+J)
 80   CONTINUE
      DO 90 I=1,N
      W(I)=XI(I)
 90   CONTINUE
      DO 95 I=1,N2
      DO 95 J=1,N1
      XI((J-1)*N2+I)=W(N1*(I-1)+J)
 95   CONTINUE
      RETURN
      END
C
      SUBROUTINE FFT235(K1,K2,K3,N,N1,N2,N3,NO2,NO3,NO5,SGN,LIST,XR,XI,
     +IRB,IRBD,IRBT,CO,SI,COD,SID,COT,SIT,WR,WI,WRD,WID,WRT,WIT,CO1,SI1,
     +CO2,SI2,CO3,SI3,W)
      DIMENSION XR(1),XI(1),IRB(1),IRBD(1),IRBT(1),CO(1),SI(1),
     +COD(1),SID(1),COT(1),SIT(1),WR(1),WI(1),WRD(1),WID(1),
     +WRT(1),WIT(1),CO1(1),SI1(1),CO2(1),SI2(1),CO3(1),
     +SI3(1),W(1)
C ***************************************************
C   THIS SUBROUTINE JOINS THE FFT2, THE FFT3, AND THE FFT5.  WHEN
C   IT IS USED, THE SUBROUTINES FFT2, FFT3, AND FFT5, MUST BE PRESENT.
C   N IS THE NUMBER OF MEASUREMENTS.  N IS OF THE FORM N=(2**K1)*
C   (3**K2)*(5**K3).  N1=2**K1, N2=3**K2, N3=5**K3.  NO2=N1/2,
C   NO3=N2/3, NO5=N3/5.  SGN=-1, IF THE FOURIER TRANSFORM IS TO BE
C   PERFORMED, AND SGN=+1 IF THE INVERSE FOURIER TRANSFORM IS TO BE
C   PERFORMED.  IF LIST=0, THE REVERSED-DIGIT LIST AND THE ASSOCIATED
C   SINES AND COSINES ARE COMPUTED.  IF LIST=1, THE LIST IS NOT
C   COMPUTED.  INSTEAD, THE LIST COMPUTED ON THE PREVIOUS CALL IS
C   USED.  XR IS THE REAL PART AND XI IS THE IMAGINARY PART OF THE
C   COMPLEX SEQUENCE TO BE TRANSFORMED.  AFTER THE FFT235 IS PERFORMED
C   XR AND XI ARE THE REAL AND IMAGINARY PARTS OF THE TRANSFORMED DATA.
C   IRB, IRBD, IRBT, CO, SI, COD, SID, COT, SIT, WR, WI, WRD, WID, WRT,
C   WIT, CO1, CO2, CO3, SI1, SI2, SI3, AND W ARE WORK VECTORS.
C ***************************************************
      TPI=6.283185307
      XN=N
      XN2=N2
      XN3=N3
      DO 65 I=1,N1
```

```
      TP1=(XL*TPI)/XN
      TP3=TP1*XN3
      DO 30 J=1,N2
      XJ=J-1
      TP2=TPI*XJ/(XN2*XN3)+TP1
      DO 10 K=1,N3
      WRT(K)=XR(N1*N2*(K-1)+N1*(J-1)+I)
      WIT(K)=XI(N1*N2*(K-1)+N1*(J-1)+I)
   10 CONTINUE
      CALL FFT5(K3,N3,NO5,SGN,LIST,WRT,WIT,IRBT,COT,SIT,CO1,SI1,CO2,SI2,
     +CO3,SI3)
      DO 20 K=1,N3
      XK=K-1
      THET=SGN*TP2*XK
      C=COS(THET)
      S=SIN(THET)
      XR(N1*N2*(K-1)+N1*(J-1)+I)=WRT(K)*C-WIT(K)*S
      XI(N1*N2*(K-1)+N1*(J-1)+I)=WIT(K)*C+WRT(K)*S
   20 CONTINUE
   30 CONTINUE
      DO 60 K=1,N3
      KM=K-1
      DO 40 J=1,N2
      WRD(J)=XR(N1*N2*KM+N1*(J-1)+I)
      WID(J)=XI(N1*N2*KM+N1*(J-1)+I)
   40 CONTINUE
      CALL FFT3(K2,N2,NO3,SGN,LIST,WRD,WID,IRBD,COD,SID)
      DO 50 J=1,N2
      XJ=J-1
      THET=SGN*TP3*XJ
      C=COS(THET)
      S=SIN(THET)
      XR(N1*N2*KM+N1*(J-1)+I)=WRD(J)*C-WID(J)*S
      XI(N1*N2*KM+N1*(J-1)+I)=WRD(J)*S+WID(J)*C
   50 CONTINUE
   60 CONTINUE
   65 CONTINUE
      DO 90 K=1,N3
      KM=K-1
      DO 80 J=1,N2
      KJ=J-1
      DO 70 I=1,N1
      WR(I)=XR(N1*N2*KM+N1*KJ+I)
      WI(I)=XI(N1*N2*KM+N1*KJ+I)
   70 CONTINUE
      CALL FFT2(K1,N1,NO2,SGN,LIST,WR,WI,IRB,CO,SI)
      DO 80 I=1,N1
      XR(N1*N2*KM+N1*KJ+I)=WR(I)
      XI(N1*N2*KM+N1*KJ+I)=WI(I)
   80 CONTINUE
   90 CONTINUE
C THE FFT COEFFICIENTS ARE UNSCRAMBLED INTO NORMAL ORDER
      DO 100 I=1,N
      W(I)=XR(I)
  100 CONTINUE
      DO 110 I=1,N1
      DO 110 J=1,N2
      MJ=J-1
      DO 110 K=1,N3
      MK=K-1
      XR((I-1)*N2*N3+N3*MJ+K)=W(N1*N2*MK+N1*MJ+I)
  110 CONTINUE
      DO 120 I=1,N
```

```
120 CONTINUE
    DO 130 I=1,N1
    DO 130 J=1,N2
    MJ=J-1
    DO 130 K=1,N3
    MK=K-1
    XI((I-1)*N2*N3+N3*MJ+K)=W(N1*N2*MK+N1*MJ+I)
130 CONTINUE
    RETURN
    END
C
    SUBROUTINE DFNORM (Z,P)
C ***********************************************************
C    SUBROUTINE COMPUTES PROBABILITY ,P, THAT A STANDARD NORMAL
C      RANDOM VARIABLE IS LESS THAN OR EQUAL TO Z
C    THE METHOD USED IS FROM: HANDBOOK OF MATH. FUNCTIONS,
C      EDITED BY ABRAMOWITZ AND STEGUN, PAGE 932, EQ.26.6.17,
C      IN THE CHAPTER BY ZELEN ABD SEVERO
C ***********************************************************
    ZZ=Z
    IF (Z.LT.0.0) ZZ=-ZZ
    T=1.0/(1.0+0.2316419*ZZ)
    E=-(ZZ*ZZ)/2.0
    IF (E.LT.-20.0) P=1.0
    IF (E.LT.-20.0) GO TO 1
    ZX=EXP(E)/2.50662828
    P=1.0-ZX*((((1.330274429*T-1.821255978)*T
    + +1.781477937)*T-0.356563782)*T+.319381530)*T
   1 IF (Z.LT.0.0) P=1.0-P
    RETURN
    END
C
    SUBROUTINE RNORM(U,Z)
C ******************************************************
C    SUBROUTINE OUTPUTS Z, GIVEN F(Z)=U FOR THE STANDARD
C      NORMAL
C ******************************************************
    P=U
    IF (U.GT.0.5) P=1.0-U
    T=SQRT(ALOG(1.0/(P*P)))
    FN=(T*0.010328+0.802853)*T+2.515517
    FD=((T*0.001308+0.189269)*T+1.432788)*T+1.0
    Z=T-FN/FD
    IF (U.LT.0.5) Z=-Z
    RETURN
    END
```

Appendix C
Output Results

```
THRESHOLD=    0.25

JJJ,V(JJJ)=   64   0.09
JJJ,V(JJJ)=   65   0.12
JJJ,V(JJJ)=   66   0.13
JJJ,V(JJJ)=   67   0.14
JJJ,V(JJJ)=   68   0.14
JJJ,V(JJJ)=   69   0.13
JJJ,V(JJJ)=   70   0.13
JJJ,V(JJJ)=   71   0.13
JJJ,V(JJJ)=   72   0.14
JJJ,V(JJJ)=   73   0.14
JJJ,V(JJJ)=   74   0.15
JJJ,V(JJJ)=   75   0.15
JJJ,V(JJJ)=   76   0.14
JJJ,V(JJJ)=   77   0.12
JJJ,V(JJJ)=   78   0.10
JJJ,V(JJJ)=   79   0.09
JJJ,V(JJJ)=   80   0.09
JJJ,V(JJJ)=   81   0.09
JJJ,V(JJJ)=   82   0.10
JJJ,V(JJJ)=   83   0.10
JJJ,V(JJJ)=   84   0.11
JJJ,V(JJJ)=   85   0.12
JJJ,V(JJJ)=   86   0.13
JJJ,V(JJJ)=   87   0.14
JJJ,V(JJJ)=   88   0.15
JJJ,V(JJJ)=   89   0.16
JJJ,V(JJJ)=   90   0.17
JJJ,V(JJJ)=   91   0.17
JJJ,V(JJJ)=   92   0.18
JJJ,V(JJJ)=   93   0.19
JJJ,V(JJJ)=   94   0.20
JJJ,V(JJJ)=   95   0.21
JJJ,V(JJJ)=   96   0.22
JJJ,V(JJJ)=   97   0.23
JJJ,V(JJJ)=   98   0.22
JJJ,V(JJJ)=   99   0.20
JJJ,V(JJJ)=   100   0.17
JJJ,V(JJJ)=   101   0.13
JJJ,V(JJJ)=   102   0.10
JJJ,V(JJJ)=   103   0.07
JJJ,V(JJJ)=   104   0.05
JJJ,V(JJJ)=   105   0.03
JJJ,V(JJJ)=   106  -0.01
JJJ,V(JJJ)=   107  -0.04
JJJ,V(JJJ)=   108  -0.08
JJJ,V(JJJ)=   109  -0.11
JJJ,V(JJJ)=   110  -0.13
```

```
JJJ,V(JJJ)=   120  -0.04
JJJ,V(JJJ)=   121  -0.05
JJJ,V(JJJ)=   122  -0.05
JJJ,V(JJJ)=   123  -0.05
JJJ,V(JJJ)=   124  -0.04
JJJ,V(JJJ)=   125  -0.04
JJJ,V(JJJ)=   126  -0.04
JJJ,V(JJJ)=   127  -0.03
JJJ,V(JJJ)=   128  -0.03
JJJ,V(JJJ)=   129  -0.02
JJJ,V(JJJ)=   130  -0.01
JJJ,V(JJJ)=   131   0.01
JJJ,V(JJJ)=   132   0.03
JJJ,V(JJJ)=   133   0.06
JJJ,V(JJJ)=   134   0.07
JJJ,V(JJJ)=   135   0.08
JJJ,V(JJJ)=   136   0.09
JJJ,V(JJJ)=   137   0.09
JJJ,V(JJJ)=   138   0.10
JJJ,V(JJJ)=   139   0.10
JJJ,V(JJJ)=   140   0.11
JJJ,V(JJJ)=   141   0.12
JJJ,V(JJJ)=   142   0.13
JJJ,V(JJJ)=   143   0.14
JJJ,V(JJJ)=   144   0.14
JJJ,V(JJJ)=   145   0.13
JJJ,V(JJJ)=   146   0.13
JJJ,V(JJJ)=   147   0.13
JJJ,V(JJJ)=   148   0.13
JJJ,V(JJJ)=   149   0.13
JJJ,V(JJJ)=   150   0.13
JJJ,V(JJJ)=   151   0.12
JJJ,V(JJJ)=   152   0.12
JJJ,V(JJJ)=   153   0.13
JJJ,V(JJJ)=   154   0.14
JJJ,V(JJJ)=   155   0.16
JJJ,V(JJJ)=   156   0.18
JJJ,V(JJJ)=   157   0.18
JJJ,V(JJJ)=   158   0.17
JJJ,V(JJJ)=   159   0.16
JJJ,V(JJJ)=   160   0.15
JJJ,V(JJJ)=   161   0.14
JJJ,V(JJJ)=   162   0.14
JJJ,V(JJJ)=   163   0.14
JJJ,V(JJJ)=   164   0.13
JJJ,V(JJJ)=   165   0.12
JJJ,V(JJJ)=   166   0.11
JJJ,V(JJJ)=   167   0.08
JJJ,V(JJJ)=   168   0.06
```

```
JJJ,V(JJJ)=  178  -0.01
JJJ,V(JJJ)=  179   0.02
JJJ,V(JJJ)=  180   0.04
JJJ,V(JJJ)=  181   0.05
JJJ,V(JJJ)=  182   0.05
JJJ,V(JJJ)=  183   0.04
JJJ,V(JJJ)=  184   0.04
JJJ,V(JJJ)=  185   0.03
JJJ,V(JJJ)=  186   0.03
JJJ,V(JJJ)=  187   0.04
JJJ,V(JJJ)=  188   0.05
JJJ,V(JJJ)=  189   0.06
JJJ,V(JJJ)=  190   0.07
JJJ,V(JJJ)=  191   0.08
JJJ,V(JJJ)=  192   0.08
JJJ,V(JJJ)=  193   0.09
JJJ,V(JJJ)=  194   0.09
JJJ,V(JJJ)=  195   0.09
JJJ,V(JJJ)=  196   0.09
JJJ,V(JJJ)=  197   0.09
JJJ,V(JJJ)=  198   0.09
JJJ,V(JJJ)=  199   0.09
JJJ,V(JJJ)=  200   0.10
JJJ,V(JJJ)=  201   0.11
JJJ,V(JJJ)=  202   0.12
JJJ,V(JJJ)=  203   0.15
JJJ,V(JJJ)=  204   0.18
JJJ,V(JJJ)=  205   0.22
JJJ,V(JJJ)=  206   0.24
JJJ,V(JJJ)=  207   0.25
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.25
JJJ,V(JJJ)=  208   0.25
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.25
JJJ,V(JJJ)=  209   0.25
JJJ,V(JJJ)=  210   0.24
JJJ,V(JJJ)=  211   0.22
JJJ,V(JJJ)=  212   0.20
JJJ,V(JJJ)=  213   0.18
JJJ,V(JJJ)=  214   0.17
JJJ,V(JJJ)=  215   0.17
JJJ,V(JJJ)=  216   0.16
JJJ,V(JJJ)=  217   0.15
JJJ,V(JJJ)=  218   0.13
JJJ,V(JJJ)=  219   0.10
JJJ,V(JJJ)=  220   0.08
JJJ,V(JJJ)=  221   0.06
JJJ,V(JJJ)=  222   0.04
JJJ,V(JJJ)=  223   0.04
JJJ,V(JJJ)=  224   0.05
```

JJJ,V(JJJ)=	234	0.12
JJJ,V(JJJ)=	235	0.12
JJJ,V(JJJ)=	236	0.11
JJJ,V(JJJ)=	237	0.11
JJJ,V(JJJ)=	238	0.12
JJJ,V(JJJ)=	239	0.13
JJJ,V(JJJ)=	240	0.14
JJJ,V(JJJ)=	241	0.14
JJJ,V(JJJ)=	242	0.13
JJJ,V(JJJ)=	243	0.13
JJJ,V(JJJ)=	244	0.12
JJJ,V(JJJ)=	245	0.11
JJJ,V(JJJ)=	246	0.11
JJJ,V(JJJ)=	247	0.11
JJJ,V(JJJ)=	248	0.11
JJJ,V(JJJ)=	249	0.12
JJJ,V(JJJ)=	250	0.13
JJJ,V(JJJ)=	251	0.14
JJJ,V(JJJ)=	252	0.15
JJJ,V(JJJ)=	253	0.16
JJJ,V(JJJ)=	254	0.17
JJJ,V(JJJ)=	255	0.17
JJJ,V(JJJ)=	256	0.15
JJJ,V(JJJ)=	257	0.14
JJJ,V(JJJ)=	258	0.12
JJJ,V(JJJ)=	259	0.12
JJJ,V(JJJ)=	260	0.12
JJJ,V(JJJ)=	261	0.12
JJJ,V(JJJ)=	262	0.13
JJJ,V(JJJ)=	263	0.14
JJJ,V(JJJ)=	264	0.15
JJJ,V(JJJ)=	265	0.16
JJJ,V(JJJ)=	266	0.15
JJJ,V(JJJ)=	267	0.13
JJJ,V(JJJ)=	268	0.10
JJJ,V(JJJ)=	269	0.06
JJJ,V(JJJ)=	270	0.03
JJJ,V(JJJ)=	271	0.01
JJJ,V(JJJ)=	272	0.00
JJJ,V(JJJ)=	273	0.00
JJJ,V(JJJ)=	274	0.00
JJJ,V(JJJ)=	275	0.00
JJJ,V(JJJ)=	276	0.00
JJJ,V(JJJ)=	277	0.01
JJJ,V(JJJ)=	278	0.01
JJJ,V(JJJ)=	279	0.01
JJJ,V(JJJ)=	280	0.00
JJJ,V(JJJ)=	281	-0.01
JJJ,V(JJJ)=	282	-0.02

JJJ,V(JJJ)=	292	0.09
JJJ,V(JJJ)=	293	0.10
JJJ,V(JJJ)=	294	0.10
JJJ,V(JJJ)=	295	0.10
JJJ,V(JJJ)=	296	0.12
JJJ,V(JJJ)=	297	0.14
JJJ,V(JJJ)=	298	0.16
JJJ,V(JJJ)=	299	0.17
JJJ,V(JJJ)=	300	0.18
JJJ,V(JJJ)=	301	0.20
JJJ,V(JJJ)=	302	0.21
JJJ,V(JJJ)=	303	0.22
JJJ,V(JJJ)=	304	0.22
JJJ,V(JJJ)=	305	0.21
JJJ,V(JJJ)=	306	0.20
JJJ,V(JJJ)=	307	0.19
JJJ,V(JJJ)=	308	0.18
JJJ,V(JJJ)=	309	0.17
JJJ,V(JJJ)=	310	0.16
JJJ,V(JJJ)=	311	0.15
JJJ,V(JJJ)=	312	0.14
JJJ,V(JJJ)=	313	0.13
JJJ,V(JJJ)=	314	0.11
JJJ,V(JJJ)=	315	0.08
JJJ,V(JJJ)=	316	0.06
JJJ,V(JJJ)=	317	0.03
JJJ,V(JJJ)=	318	0.01
JJJ,V(JJJ)=	319	-0.01
JJJ,V(JJJ)=	320	-0.01
JJJ,V(JJJ)=	321	0.00
JJJ,V(JJJ)=	322	0.00
JJJ,V(JJJ)=	323	0.01
JJJ,V(JJJ)=	324	0.01
JJJ,V(JJJ)=	325	0.01
JJJ,V(JJJ)=	326	0.01
JJJ,V(JJJ)=	327	0.00
JJJ,V(JJJ)=	328	-0.02
JJJ,V(JJJ)=	329	-0.03
JJJ,V(JJJ)=	330	-0.04
JJJ,V(JJJ)=	331	-0.06
JJJ,V(JJJ)=	332	-0.08
JJJ,V(JJJ)=	333	-0.10
JJJ,V(JJJ)=	334	-0.11
JJJ,V(JJJ)=	335	-0.13
JJJ,V(JJJ)=	336	-0.14
JJJ,V(JJJ)=	337	-0.15
JJJ,V(JJJ)=	338	-0.16
JJJ,V(JJJ)=	339	-0.17
JJJ,V(JJJ)=	340	-0.18

```
JJJ,V(JJJ)=  350  -0.03
JJJ,V(JJJ)=  351  -0.03
JJJ,V(JJJ)=  352  -0.03
JJJ,V(JJJ)=  353  -0.03
JJJ,V(JJJ)=  354  -0.03
JJJ,V(JJJ)=  355  -0.02
JJJ,V(JJJ)=  356  -0.02
JJJ,V(JJJ)=  357  -0.02
JJJ,V(JJJ)=  358  -0.01
JJJ,V(JJJ)=  359   0.00
JJJ,V(JJJ)=  360   0.01
JJJ,V(JJJ)=  361   0.02
JJJ,V(JJJ)=  362   0.03
JJJ,V(JJJ)=  363   0.05
JJJ,V(JJJ)=  364   0.06
JJJ,V(JJJ)=  365   0.06
JJJ,V(JJJ)=  366   0.06
JJJ,V(JJJ)=  367   0.06
JJJ,V(JJJ)=  368   0.06
JJJ,V(JJJ)=  369   0.07
JJJ,V(JJJ)=  370   0.08
JJJ,V(JJJ)=  371   0.09
JJJ,V(JJJ)=  372   0.11
JJJ,V(JJJ)=  373   0.12
JJJ,V(JJJ)=  374   0.11
JJJ,V(JJJ)=  375   0.10
JJJ,V(JJJ)=  376   0.09
JJJ,V(JJJ)=  377   0.07
JJJ,V(JJJ)=  378   0.06
JJJ,V(JJJ)=  379   0.06
JJJ,V(JJJ)=  380   0.05
JJJ,V(JJJ)=  381   0.04
JJJ,V(JJJ)=  382   0.03
JJJ,V(JJJ)=  383   0.02
JJJ,V(JJJ)=  384   0.01
JJJ,V(JJJ)=  385  -0.01
JJJ,V(JJJ)=  386  -0.03
JJJ,V(JJJ)=  387  -0.05
JJJ,V(JJJ)=  388  -0.07
JJJ,V(JJJ)=  389  -0.08
JJJ,V(JJJ)=  390  -0.10
JJJ,V(JJJ)=  391  -0.10
JJJ,V(JJJ)=  392  -0.10
JJJ,V(JJJ)=  393  -0.09
JJJ,V(JJJ)=  394  -0.08
JJJ,V(JJJ)=  395  -0.07
JJJ,V(JJJ)=  396  -0.05
JJJ,V(JJJ)=  397  -0.04
JJJ,V(JJJ)=  398  -0.03
```

```
JJJ,V(JJJ)=  408  -0.13
JJJ,V(JJJ)=  409  -0.16
JJJ,V(JJJ)=  410  -0.18
JJJ,V(JJJ)=  411  -0.19
JJJ,V(JJJ)=  412  -0.20
JJJ,V(JJJ)=  413  -0.22
JJJ,V(JJJ)=  414  -0.22
JJJ,V(JJJ)=  415  -0.22
JJJ,V(JJJ)=  416  -0.21
JJJ,V(JJJ)=  417  -0.19
JJJ,V(JJJ)=  418  -0.17
JJJ,V(JJJ)=  419  -0.14
JJJ,V(JJJ)=  420  -0.11
JJJ,V(JJJ)=  421  -0.08
JJJ,V(JJJ)=  422  -0.05
JJJ,V(JJJ)=  423  -0.03
JJJ,V(JJJ)=  424  -0.03
JJJ,V(JJJ)=  425  -0.02
JJJ,V(JJJ)=  426  -0.02
JJJ,V(JJJ)=  427  -0.01
JJJ,V(JJJ)=  428   0.00
JJJ,V(JJJ)=  429   0.01
JJJ,V(JJJ)=  430   0.01
JJJ,V(JJJ)=  431   0.01
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JJJ,V(JJJ)= 1094 -0.09

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EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.28
AR SHUTDOWN STEP AT 1175
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EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.27
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JJJ,V(JJJ)= 1188 0.24
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EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.27
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EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.30
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EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.32
KRIGING SHUTDOWN STEP AT 1198
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EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.34
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KRIGING SHUTDOWN STEP AT   1202
AR SHUTDOWN STEP AT   1202
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EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.27
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EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.27
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EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.25
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JJJ,V(JJJ)=  1565   0.00
JJJ,V(JJJ)=  1566   0.00
JJJ,V(JJJ)=  1567   0.00
JJJ,V(JJJ)=  1568   0.00
JJJ,V(JJJ)=  1569   0.02
JJJ,V(JJJ)=  1570   0.03
JJJ,V(JJJ)=  1571   0.06
JJJ,V(JJJ)=  1572   0.08
JJJ,V(JJJ)=  1573   0.11
JJJ,V(JJJ)=  1574   0.13
JJJ,V(JJJ)=  1575   0.17
JJJ,V(JJJ)=  1576   0.20
JJJ,V(JJJ)=  1577   0.22
JJJ,V(JJJ)=  1578   0.23
JJJ,V(JJJ)=  1579   0.22
JJJ,V(JJJ)=  1580   0.20
JJJ,V(JJJ)=  1581   0.17
JJJ,V(JJJ)=  1582   0.14
JJJ,V(JJJ)=  1583   0.11
JJJ,V(JJJ)=  1584   0.10
JJJ,V(JJJ)=  1585   0.09
JJJ,V(JJJ)=  1586   0.10
JJJ,V(JJJ)=  1587   0.11
JJJ,V(JJJ)=  1588   0.13
JJJ,V(JJJ)=  1589   0.14
JJJ,V(JJJ)=  1590   0.14
JJJ,V(JJJ)=  1591   0.14
JJJ,V(JJJ)=  1592   0.14
JJJ,V(JJJ)=  1593   0.14
```

```
KRIGING SHUTDOWN STEP AT   1600
AR SHUTDOWN STEP AT   1600
JJJ,V(JJJ)=  1601    0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.30
KRIGING SHUTDOWN STEP AT   1601
AR SHUTDOWN STEP AT   1601
JJJ,V(JJJ)=  1602    0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.30
KRIGING SHUTDOWN STEP AT   1602
AR SHUTDOWN STEP AT   1602
JJJ,V(JJJ)=  1603    0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.29
KRIGING SHUTDOWN STEP AT   1603
AR SHUTDOWN STEP AT   1603
JJJ,V(JJJ)=  1604    0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.27
KRIGING SHUTDOWN STEP AT   1604
AR SHUTDOWN STEP AT   1604
JJJ,V(JJJ)=  1605    0.24
JJJ,V(JJJ)=  1606    0.21
JJJ,V(JJJ)=  1607    0.18
JJJ,V(JJJ)=  1608    0.15
JJJ,V(JJJ)=  1609    0.13
JJJ,V(JJJ)=  1610    0.12
JJJ,V(JJJ)=  1611    0.12
JJJ,V(JJJ)=  1612    0.13
JJJ,V(JJJ)=  1613    0.15
JJJ,V(JJJ)=  1614    0.16
JJJ,V(JJJ)=  1615    0.16
JJJ,V(JJJ)=  1616    0.16
JJJ,V(JJJ)=  1617    0.15
JJJ,V(JJJ)=  1618    0.14
JJJ,V(JJJ)=  1619    0.13
JJJ,V(JJJ)=  1620    0.13
JJJ,V(JJJ)=  1621    0.14
JJJ,V(JJJ)=  1622    0.17
JJJ,V(JJJ)=  1623    0.20
JJJ,V(JJJ)=  1624    0.23
JJJ,V(JJJ)=  1625    0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.27
JJJ,V(JJJ)=  1626    0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.29
KRIGING SHUTDOWN STEP AT   1626
AR SHUTDOWN STEP AT   1626
JJJ,V(JJJ)=  1627    0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.31
KRIGING SHUTDOWN STEP AT   1627
AR SHUTDOWN STEP AT   1627
JJJ,V(JJJ)=  1628    0.31
```

```
KRIGING SHUTDOWN STEP AT   1630
JJJ,V(JJJ)=  1631   0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.27
KRIGING SHUTDOWN STEP AT   1631
JJJ,V(JJJ)=  1632   0.25
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.25
JJJ,V(JJJ)=  1633   0.24
JJJ,V(JJJ)=  1634   0.23
JJJ,V(JJJ)=  1635   0.23
JJJ,V(JJJ)=  1636   0.25
JJJ,V(JJJ)=  1637   0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.27
KRIGING SHUTDOWN STEP AT   1637
AR SHUTDOWN STEP AT   1637
JJJ,V(JJJ)=  1638   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.29
KRIGING SHUTDOWN STEP AT   1638
AR SHUTDOWN STEP AT   1638
JJJ,V(JJJ)=  1639   0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.31
KRIGING SHUTDOWN STEP AT   1639
AR SHUTDOWN STEP AT   1639
JJJ,V(JJJ)=  1640   0.32
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.32
KRIGING SHUTDOWN STEP AT   1640
AR SHUTDOWN STEP AT   1640
JJJ,V(JJJ)=  1641   0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.31
KRIGING SHUTDOWN STEP AT   1641
AR SHUTDOWN STEP AT   1641
JJJ,V(JJJ)=  1642   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.30
KRIGING SHUTDOWN STEP AT   1642
AR SHUTDOWN STEP AT   1642
JJJ,V(JJJ)=  1643   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.29
KRIGING SHUTDOWN STEP AT   1643
AR SHUTDOWN STEP AT   1643
JJJ,V(JJJ)=  1644   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.29
KRIGING SHUTDOWN STEP AT   1644
AR SHUTDOWN STEP AT   1644
JJJ,V(JJJ)=  1645   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.30
KRIGING SHUTDOWN STEP AT   1645
AR SHUTDOWN STEP AT   1645
JJJ,V(JJJ)=  1646   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.30
KRIGING SHUTDOWN STEP AT   1646
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JJJ,V(JJJ)= 1649 0.36
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.36
KRIGING SHUTDOWN STEP AT 1649
AR SHUTDOWN STEP AT 1649
JJJ,V(JJJ)= 1650 0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.40
KRIGING SHUTDOWN STEP AT 1650
AR SHUTDOWN STEP AT 1650
JJJ,V(JJJ)= 1651 0.43
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.43
KRIGING SHUTDOWN STEP AT 1651
AR SHUTDOWN STEP AT 1651
JJJ,V(JJJ)= 1652 0.45
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.45
KRIGING SHUTDOWN STEP AT 1652
AR SHUTDOWN STEP AT 1652
JJJ,V(JJJ)= 1653 0.46
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.46
KRIGING SHUTDOWN STEP AT 1653
AR SHUTDOWN STEP AT 1653
JJJ,V(JJJ)= 1654 0.45
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.45
KRIGING SHUTDOWN STEP AT 1654
AR SHUTDOWN STEP AT 1654
JJJ,V(JJJ)= 1655 0.43
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.43
KRIGING SHUTDOWN STEP AT 1655
AR SHUTDOWN STEP AT 1655
JJJ,V(JJJ)= 1656 0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.40
KRIGING SHUTDOWN STEP AT 1656
AR SHUTDOWN STEP AT 1656
JJJ,V(JJJ)= 1657 0.37
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.37
KRIGING SHUTDOWN STEP AT 1657
AR SHUTDOWN STEP AT 1657
JJJ,V(JJJ)= 1658 0.33
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.33
KRIGING SHUTDOWN STEP AT 1658
AR SHUTDOWN STEP AT 1658
JJJ,V(JJJ)= 1659 0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.30
KRIGING SHUTDOWN STEP AT 1659
AR SHUTDOWN STEP AT 1659
JJJ,V(JJJ)= 1660 0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE= 0.29
KRIGING SHUTDOWN STEP AT 1660
AR SHUTDOWN STEP AT 1660
JJJ,V(JJJ)= 1661 0.30
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KRIGING SHUTDOWN STEP AT   1663
AR SHUTDOWN STEP AT   1663
JJJ,V(JJJ)=  1664   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.35
KRIGING SHUTDOWN STEP AT   1664
AR SHUTDOWN STEP AT   1664
JJJ,V(JJJ)=  1665   0.36
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.36
KRIGING SHUTDOWN STEP AT   1665
AR SHUTDOWN STEP AT   1665
JJJ,V(JJJ)=  1666   0.36
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.36
KRIGING SHUTDOWN STEP AT   1666
AR SHUTDOWN STEP AT   1666
JJJ,V(JJJ)=  1667   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.35
KRIGING SHUTDOWN STEP AT   1667
AR SHUTDOWN STEP AT   1667
JJJ,V(JJJ)=  1668   0.33
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.33
KRIGING SHUTDOWN STEP AT   1668
AR SHUTDOWN STEP AT   1668
JJJ,V(JJJ)=  1669   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.30
KRIGING SHUTDOWN STEP AT   1669
AR SHUTDOWN STEP AT   1669
JJJ,V(JJJ)=  1670   0.28
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.28
KRIGING SHUTDOWN STEP AT   1670
AR SHUTDOWN STEP AT   1670
JJJ,V(JJJ)=  1671   0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.27
KRIGING SHUTDOWN STEP AT   1671
AR SHUTDOWN STEP AT   1671
JJJ,V(JJJ)=  1672   0.26
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.26
KRIGING SHUTDOWN STEP AT   1672
AR SHUTDOWN STEP AT   1672
JJJ,V(JJJ)=  1673   0.28
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.28
KRIGING SHUTDOWN STEP AT   1673
AR SHUTDOWN STEP AT   1673
JJJ,V(JJJ)=  1674   0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.31
KRIGING SHUTDOWN STEP AT   1674
AR SHUTDOWN STEP AT   1674
JJJ,V(JJJ)=  1675   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=      0.35
KRIGING SHUTDOWN STEP AT   1675
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JJJ,V(JJJ)= 1678   0.44
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.44
KRIGING SHUTDOWN STEP AT   1678
AR SHUTDOWN STEP AT   1678
JJJ,V(JJJ)= 1679   0.43
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.43
KRIGING SHUTDOWN STEP AT   1679
AR SHUTDOWN STEP AT   1679
JJJ,V(JJJ)= 1680   0.39
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.39
AR SHUTDOWN STEP AT   1680
JJJ,V(JJJ)= 1681   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.35
KRIGING SHUTDOWN STEP AT   1681
AR SHUTDOWN STEP AT   1681
JJJ,V(JJJ)= 1682   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.29
KRIGING SHUTDOWN STEP AT   1682
AR SHUTDOWN STEP AT   1682
JJJ,V(JJJ)= 1683   0.25
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.25
KRIGING SHUTDOWN STEP AT   1683
AR SHUTDOWN STEP AT   1683
JJJ,V(JJJ)= 1684   0.23
JJJ,V(JJJ)= 1685   0.23
JJJ,V(JJJ)= 1686   0.24
JJJ,V(JJJ)= 1687   0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.27
AR SHUTDOWN STEP AT   1687
JJJ,V(JJJ)= 1688   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.30
KRIGING SHUTDOWN STEP AT   1688
AR SHUTDOWN STEP AT   1688
JJJ,V(JJJ)= 1689   0.34
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.34
KRIGING SHUTDOWN STEP AT   1689
AR SHUTDOWN STEP AT   1689
JJJ,V(JJJ)= 1690   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.35
KRIGING SHUTDOWN STEP AT   1690
AR SHUTDOWN STEP AT   1690
JJJ,V(JJJ)= 1691   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.35
KRIGING SHUTDOWN STEP AT   1691
AR SHUTDOWN STEP AT   1691
JJJ,V(JJJ)= 1692   0.34
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.34
KRIGING SHUTDOWN STEP AT   1692
AR SHUTDOWN STEP AT   1692
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EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.25
JJJ,V(JJJ)=  1699   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.29
JJJ,V(JJJ)=  1700   0.33
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.33
JJJ,V(JJJ)=  1701   0.37
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.37
JJJ,V(JJJ)=  1702   0.39
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.39
JJJ,V(JJJ)=  1703   0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.40
JJJ,V(JJJ)=  1704   0.38
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.38
JJJ,V(JJJ)=  1705   0.34
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.34
JJJ,V(JJJ)=  1706   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.29
JJJ,V(JJJ)=  1707   0.25
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.25
JJJ,V(JJJ)=  1708   0.23
JJJ,V(JJJ)=  1709   0.22
JJJ,V(JJJ)=  1710   0.23
JJJ,V(JJJ)=  1711   0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.27
JJJ,V(JJJ)=  1712   0.32
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.32
JJJ,V(JJJ)=  1713   0.37
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.37
JJJ,V(JJJ)=  1714   0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.40
JJJ,V(JJJ)=  1715   0.41
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.41
JJJ,V(JJJ)=  1716   0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.40
JJJ,V(JJJ)=  1717   0.37
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.37
JJJ,V(JJJ)=  1718   0.35
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.35
JJJ,V(JJJ)=  1719   0.33
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.33
JJJ,V(JJJ)=  1720   0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.31
JJJ,V(JJJ)=  1721   0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.31
KRIGING SHUTDOWN STEP AT   1721
JJJ,V(JJJ)=  1722   0.32
EXCEEDANCE OF THRESHOLD FOUND, VALUE=    0.32
KRIGING SHUTDOWN STEP AT   1722
JJJ,V(JJJ)=  1723   0.33
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KRIGING SHUTDOWN STEP AT   1725
AR SHUTDOWN STEP AT   1725
JJJ,V(JJJ)=  1726   0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.40
KRIGING SHUTDOWN STEP AT   1726
AR SHUTDOWN STEP AT   1726
JJJ,V(JJJ)=  1727   0.41
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0 41
KRIGING SHUTDOWN STEP AT   1727
AR SHUTDOWN STEP AT   1727
JJJ,V(JJJ)=  1728   0.40
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.40
KRIGING SHUTDOWN STEP AT   1728
AR SHUTDOWN STEP AT   1728
JJJ,V(JJJ)=  1729   0.36
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.36
KRIGING SHUTDOWN STEP AT   1729
AR SHUTDOWN STEP AT   1729
JJJ,V(JJJ)=  1730   0.31
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.31
AR SHUTDOWN STEP AT   1730
JJJ,V(JJJ)=  1731   0.25
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.25
JJJ,V(JJJ)=  1732   0.21
JJJ,V(JJJ)=  1733   0.18
JJJ,V(JJJ)=  1734   0.17
JJJ,V(JJJ)=  1735   0.19
JJJ,V(JJJ)=  1736   0.23
JJJ,V(JJJ)=  1737   0.27
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.27
KRIGING SHUTDOWN STEP AT   1737
AR SHUTDOWN STEP AT   1737
JJJ,V(JJJ)=  1738   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.30
KRIGING SHUTDOWN STEP AT   1738
AR SHUTDOWN STEP AT   1738
JJJ,V(JJJ)=  1739   0.30
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.30
KRIGING SHUTDOWN STEP AT   1739
AR SHUTDOWN STEP AT   1739
JJJ,V(JJJ)=  1740   0.29
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.29
AR SHUTDOWN STEP AT   1740
JJJ,V(JJJ)=  1741   0.26
EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.26
AR SHUTDOWN STEP AT   1741
JJJ,V(JJJ)=  1742   0.22
JJJ,V(JJJ)=  1743   0.18
JJJ,V(JJJ)=  1744   0.15
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EXCEEDANCE OF THRESHOLD FOUND, VALUE=     0.26
JJJ,V(JJJ)=  1753   0.23
JJJ,V(JJJ)=  1754   0 19
JJJ,V(JJJ)=  1755   0.14
JJJ,V(JJJ)=  1756   0.09
JJJ,V(JJJ)=  1757   0.06
JJJ,V(JJJ)=  1758   0.04
JJJ,V(JJJ)=  1759   0.04
JJJ,V(JJJ)=  1760   0.06
JJJ,V(JJJ)=  1761   0.09
JJJ,V(JJJ)=  1762   0.12
JJJ,V(JJJ)=  1763   0.13
JJJ,V(JJJ)=  1764   0.13
JJJ,V(JJJ)=  1765   0.11
JJJ,V(JJJ)=  1766   0.09
JJJ,V(JJJ)=  1767   0.06
JJJ,V(JJJ)=  1768   0.04
JJJ,V(JJJ)=  1769   0.04
JJJ,V(JJJ)=  1770   0.05
JJJ,V(JJJ)=  1771   0.07
JJJ,V(JJJ)=  1772   0.09
JJJ,V(JJJ)=  1773   0.12
JJJ,V(JJJ)=  1774   0.15
JJJ,V(JJJ)=  1775   0.18
JJJ,V(JJJ)=  1776   0.18
JJJ,V(JJJ)=  1777   0.18
JJJ,V(JJJ)=  1778   0.16
JJJ,V(JJJ)=  1779   0.13
JJJ,V(JJJ)=  1780   0.10
JJJ,V(JJJ)=  1781   0.06
JJJ,V(JJJ)=  1782   0.04
JJJ,V(JJJ)=  1783   0.03
JJJ,V(JJJ)=  1784   0.04
JJJ,V(JJJ)=  1785   0.06
JJJ,V(JJJ)=  1786   0.09
JJJ,V(JJJ)=  1787   0.11
JJJ,V(JJJ)=  1788   0.13
JJJ,V(JJJ)=  1789   0.13
JJJ,V(JJJ)=  1790   0.12
JJJ,V(JJJ)=  1791   0.09
JJJ,V(JJJ)=  1792   0.06
JJJ,V(JJJ)=  1793   0.04
JJJ,V(JJJ)=  1794   0.03
JJJ,V(JJJ)=  1795   0.04
JJJ,V(JJJ)=  1796   0.07
JJJ,V(JJJ)=  1797   0.09
JJJ,V(JJJ)=  1798   0.12
JJJ,V(JJJ)=  1799   0.15
JJJ,V(JJJ)=  1800   0.17
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JJJ,V(JJJ)= 1810 0.16
JJJ,V(JJJ)= 1811 0.18
JJJ,V(JJJ)= 1812 0.20
JJJ,V(JJJ)= 1813 0.20
JJJ,V(JJJ)= 1814 0.19
JJJ,V(JJJ)= 1815 0.16
JJJ,V(JJJ)= 1816 0.12
JJJ,V(JJJ)= 1817 0.08
JJJ,V(JJJ)= 1818 0.06
JJJ,V(JJJ)= 1819 0.04
JJJ,V(JJJ)= 1820 0.03
JJJ,V(JJJ)= 1821 0.04
JJJ,V(JJJ)= 1822 0.04
JJJ,V(JJJ)= 1823 0.05
JJJ,V(JJJ)= 1824 0.06
JJJ,V(JJJ)= 1825 0.06
JJJ,V(JJJ)= 1826 0.06
JJJ,V(JJJ)= 1827 0.04
JJJ,V(JJJ)= 1828 0.02
JJJ,V(JJJ)= 1829 0.00
JJJ,V(JJJ)= 1830 -0.02
JJJ,V(JJJ)= 1831 -0.04
JJJ,V(JJJ)= 1832 -0.04
JJJ,V(JJJ)= 1833 -0.04
JJJ,V(JJJ)= 1834 -0.03
JJJ,V(JJJ)= 1835 -0.01
JJJ,V(JJJ)= 1836 0.00
JJJ,V(JJJ)= 1837 0.02
JJJ,V(JJJ)= 1838 0.03
JJJ,V(JJJ)= 1839 0.03
JJJ,V(JJJ)= 1840 0.01
JJJ,V(JJJ)= 1841 -0.01
JJJ,V(JJJ)= 1842 -0.03
JJJ,V(JJJ)= 1843 -0.04
JJJ,V(JJJ)= 1844 -0.04
JJJ,V(JJJ)= 1845 -0.04
JJJ,V(JJJ)= 1846 -0.03
JJJ,V(JJJ)= 1847 -0.01
JJJ,V(JJJ)= 1848 0.00
JJJ,V(JJJ)= 1849 0.01
JJJ,V(JJJ)= 1850 0.02
JJJ,V(JJJ)= 1851 0.02
JJJ,V(JJJ)= 1852 0.01
JJJ,V(JJJ)= 1853 0.00
JJJ,V(JJJ)= 1854 -0.01
JJJ,V(JJJ)= 1855 -0.02
JJJ,V(JJJ)= 1856 -0.02
JJJ,V(JJJ)= 1857 -0.02
JJJ,V(JJJ)= 1858 0.00
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JJJ,V(JJJ)=  1868   0.16
JJJ,V(JJJ)=  1869   0.17
JJJ,V(JJJ)=  1870   0.19
JJJ,V(JJJ)=  1871   0.20
JJJ,V(JJJ)=  1872   0.21
JJJ,V(JJJ)=  1873   0.22
JJJ,V(JJJ)=  1874   0.23
JJJ,V(JJJ)=  1875   0.23
JJJ,V(JJJ)=  1876   0.23
JJJ,V(JJJ)=  1877   0.21
JJJ,V(JJJ)=  1878   0.20
JJJ,V(JJJ)=  1879   0.18
JJJ,V(JJJ)=  1880   0.17
JJJ,V(JJJ)=  1881   0.16
JJJ,V(JJJ)=  1882   0.15
JJJ,V(JJJ)=  1883   0.15
JJJ,V(JJJ)=  1884   0.15
JJJ,V(JJJ)=  1885   0.16
JJJ,V(JJJ)=  1886   0.17
JJJ,V(JJJ)=  1887   0.18
JJJ,V(JJJ)=  1888   0.17
JJJ,V(JJJ)=  1889   0.16
JJJ,V(JJJ)=  1890   0.14
JJJ,V(JJJ)=  1891   0.12
JJJ,V(JJJ)=  1892   0.10
JJJ,V(JJJ)=  1893   0.09
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JJJ,V(JJJ)=  1915   0.00
JJJ,V(JJJ)=  1916  -0.02
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KRIGING SHUTDOWN STEP AT   1954
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Two statistical techniques for making rapid, short-term predictions of the harmonically determined tidal residual are presented. The techniques are applied to an Automated Real-Time Tidal Elevation System (ARTTES) to improve short-term tide prediction. The techniques investigated are conventionally called "Kriging" and autoregression. The autoregressive method is based on the Yule-Walker equations for stationary time series. Evaluations using both synthetic and prototype data indicate that although Kriging has a slightly larger root mean square error than the autoregressive technique, the autoregressive technique shows bias, while Kriging gives an unbiased estimate.

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